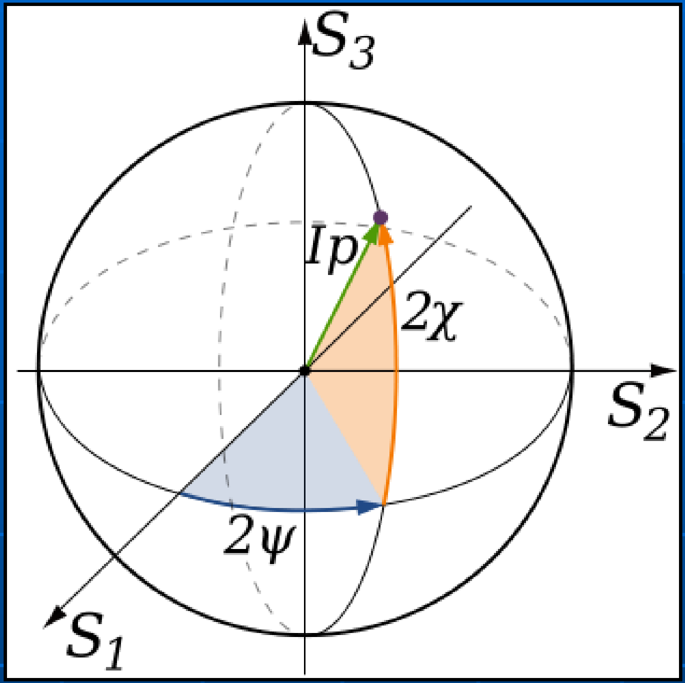
Polarization & Cosmic Magnetic Fields

Rainer Beck, MPIfR

Polarization
is an extra dimension
in the phase space
of observations

Stokes parameters

Poincaré sphere



I

$Q = I\ p\ \cos 2\psi \cos 2\chi$

$U = I\ p\ \sin 2\psi \cos 2\chi$

$V = I\ p\ \sin 2\chi$

p: degree of polarization

ψ: Angle of linear polarization

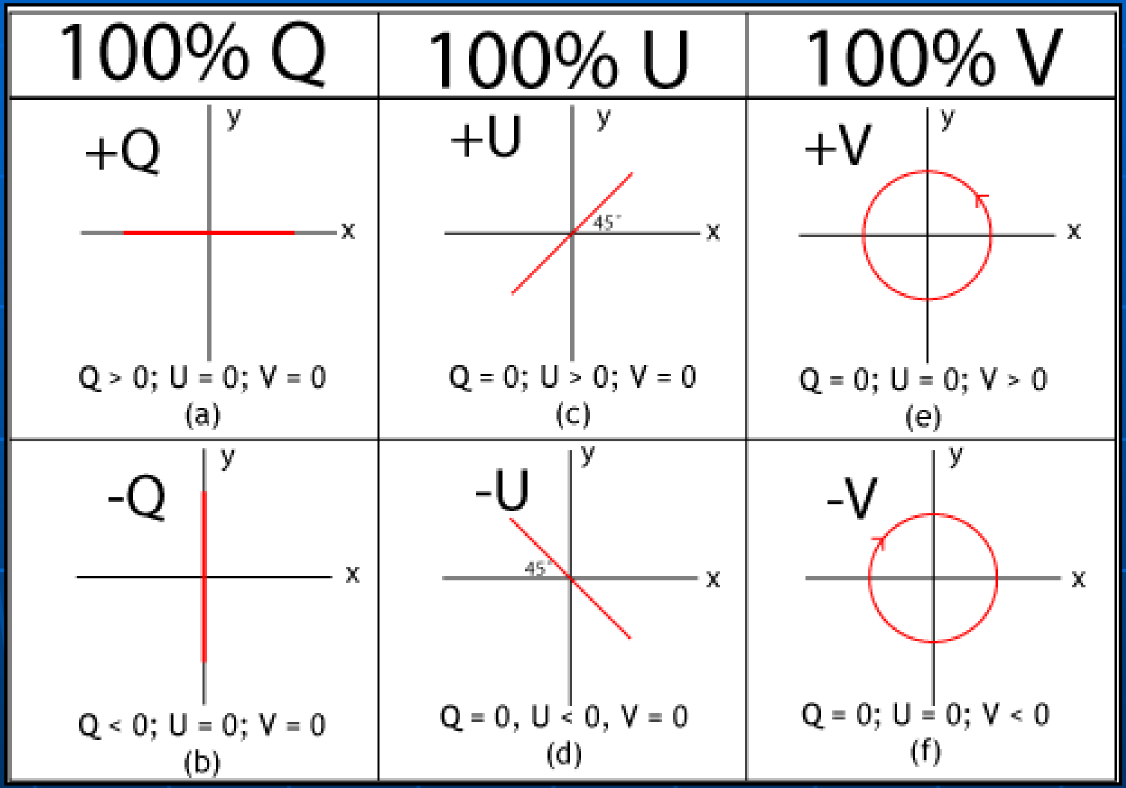
$\psi = 0.5 \arctan (U/Q)$

χ: "Angle" of circular polarization

The projected polarization plane is NOT a vector !

(no direction, only orientation)

Stokes parameters



$\psi = 0.5 \arctan (U/Q)$

Astronomy: x points towards the north

Complex (linear) polarization

$$P = p \exp (2 i \psi)$$

Stokes vector

The Stokes parameters can be arranged in a Stokes vector:

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} E_{0x}^2 + E_{0y}^2 \\ E_{0x}^2 - E_{0y}^2 \\ 2E_{0x}E_{0y}\cos\varepsilon \\ 2E_{0x}E_{0y}\sin\varepsilon \end{pmatrix} = \begin{pmatrix} \text{intensity} \\ I(0°) - I(90°) \\ I(45°) - I(135°) \\ I(RCP) - I(LCP) \end{pmatrix}$$

Linear polarization:	$Q \neq 0, U \neq 0, V = 0$
Circular polarization:	$Q = 0, U = 0, V \neq 0$
Fully polarized light:	$I^2 = Q^2 + U^2 + V^2$
Partially polarized light:	$I^2 > Q^2 + U^2 + V^2$
Unpolarized light:	$Q = U = V = 0$

Mueller matrix

If light is represented by Stokes vectors, optical components are then described with Mueller matrices:

[output light] = [Mueller matrix] [input light]

$$\begin{pmatrix} I' \\ Q' \\ U' \\ V' \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

Mueller matrices of polarizers

Linear (±Q) polarizer at 0º:

$$0.5\begin{pmatrix} 1 & \pm1 & 0 & 0 \\ \pm1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Linear (±U) polarizer at 0º :

$$0.5\begin{pmatrix} 1 & 0 & \pm1 & 0 \\ 0 & 0 & 0 & 0 \\ \pm1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

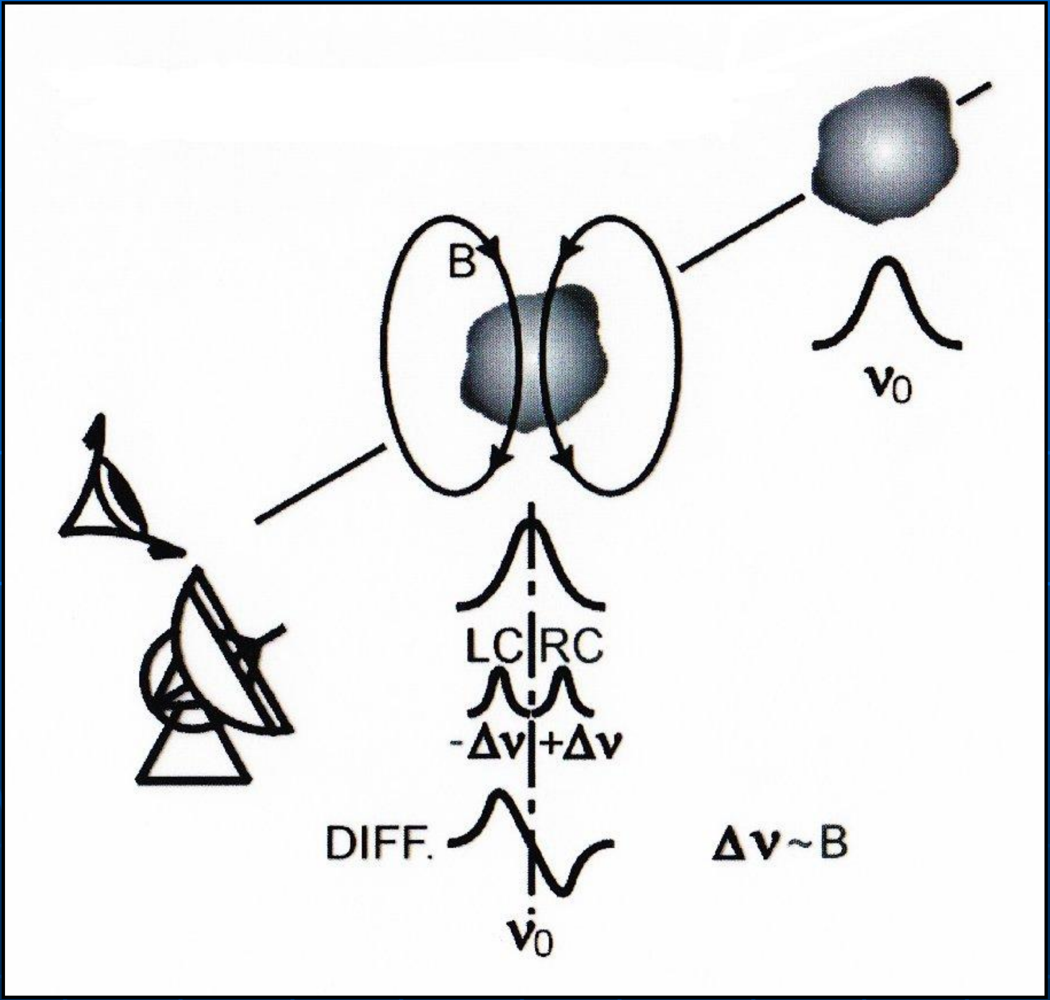
Circular (±V) polarizer at 0º :

$$0.5\begin{pmatrix} 1 & 0 & 0 & \pm1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \pm1 & 0 & 0 & 1 \end{pmatrix}$$

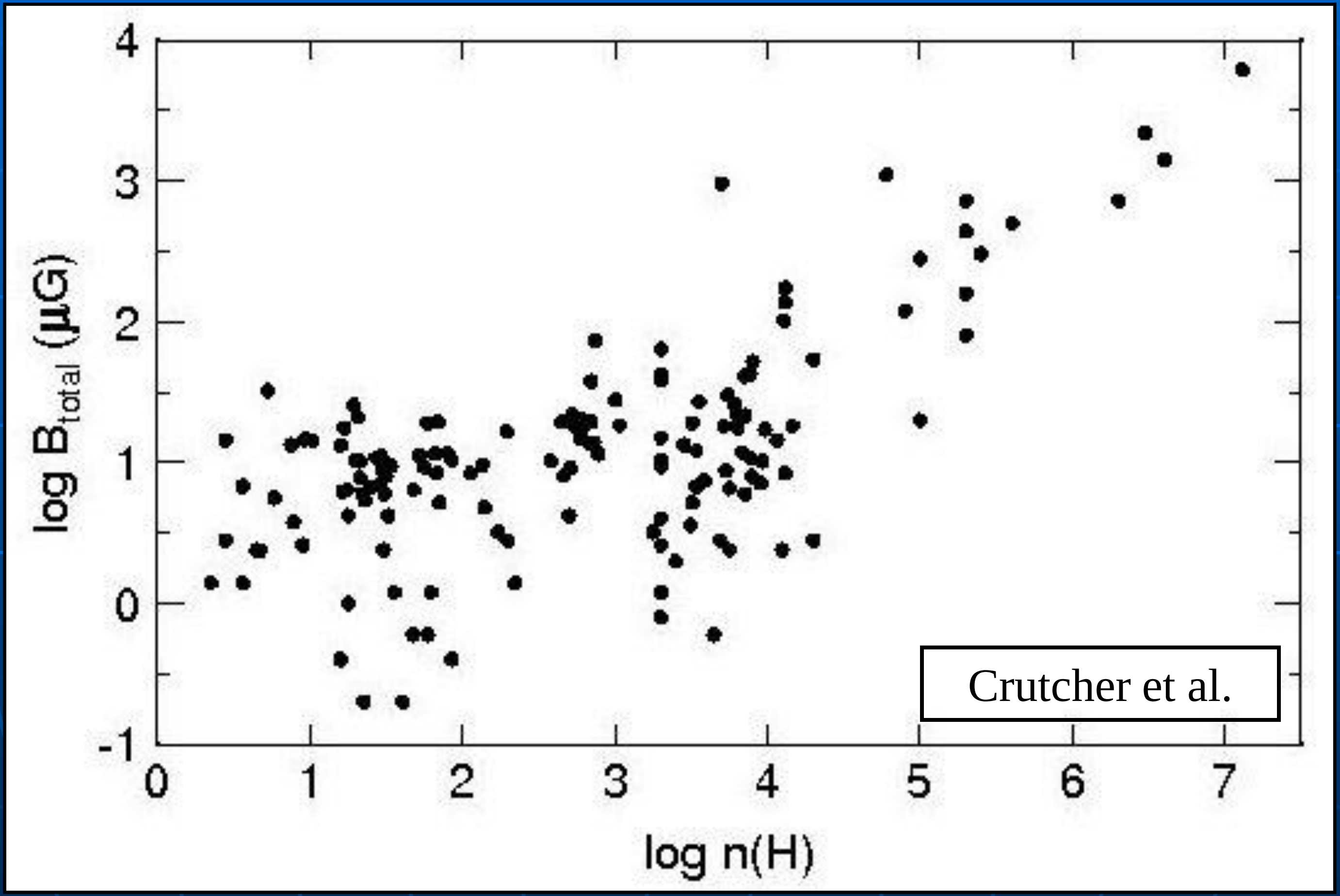
Tools to study magnetic fields

- Zeeman effect:
  Strength and sign of ordered $B_{\parallel}$
- Optical / infrared / submm polarization by dust grains:
  Structure of ordered $B_{\perp}$
- Total synchrotron intensity:
  Strength of total $B_{\perp}$
- Polarized synchrotron intensity:
  Strength and structure of ordered $B_{\perp}$
- Faraday rotation:
  Strength and sign of ordered $B_{\parallel}$
- Faraday depolarization:
  Strength and scale of turbulent fields

Zeeman effect

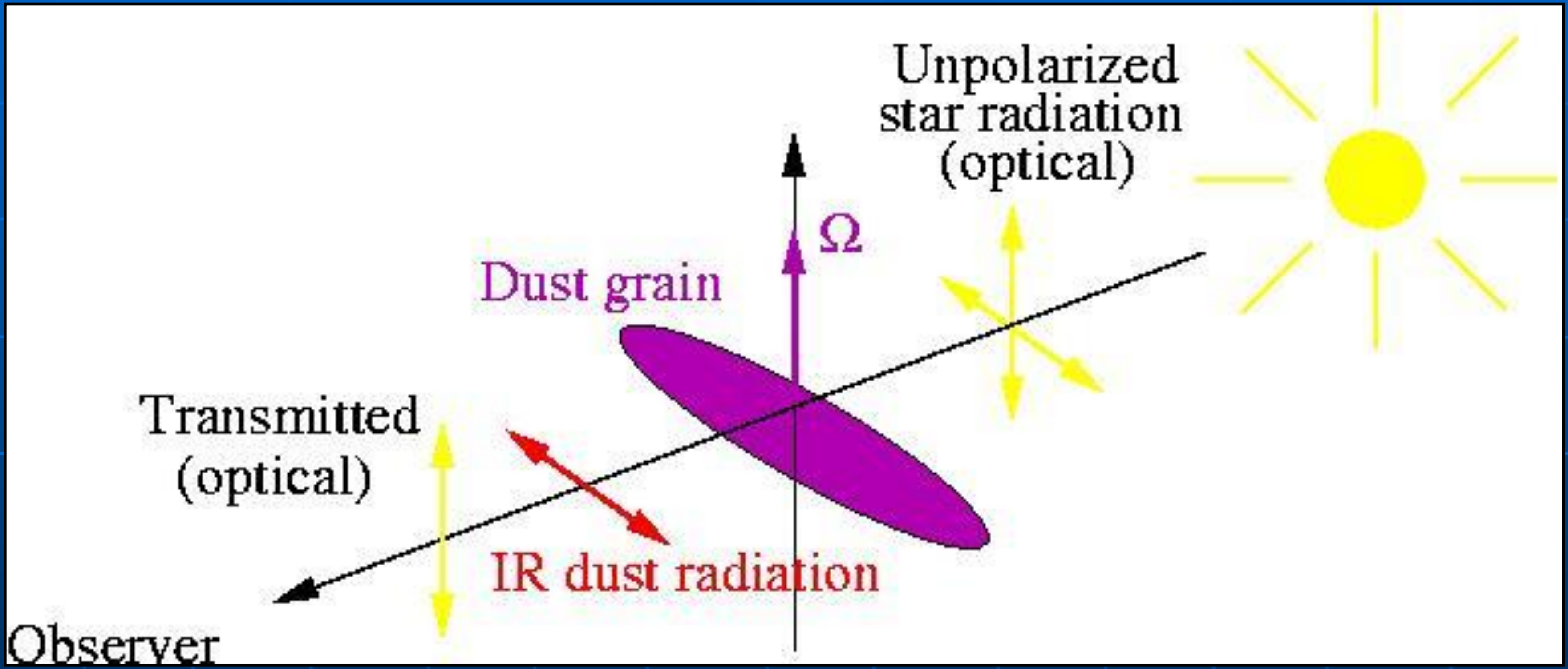


Zeeman effect:
Field strengths ($B_{\|}$) in Milky Way clouds

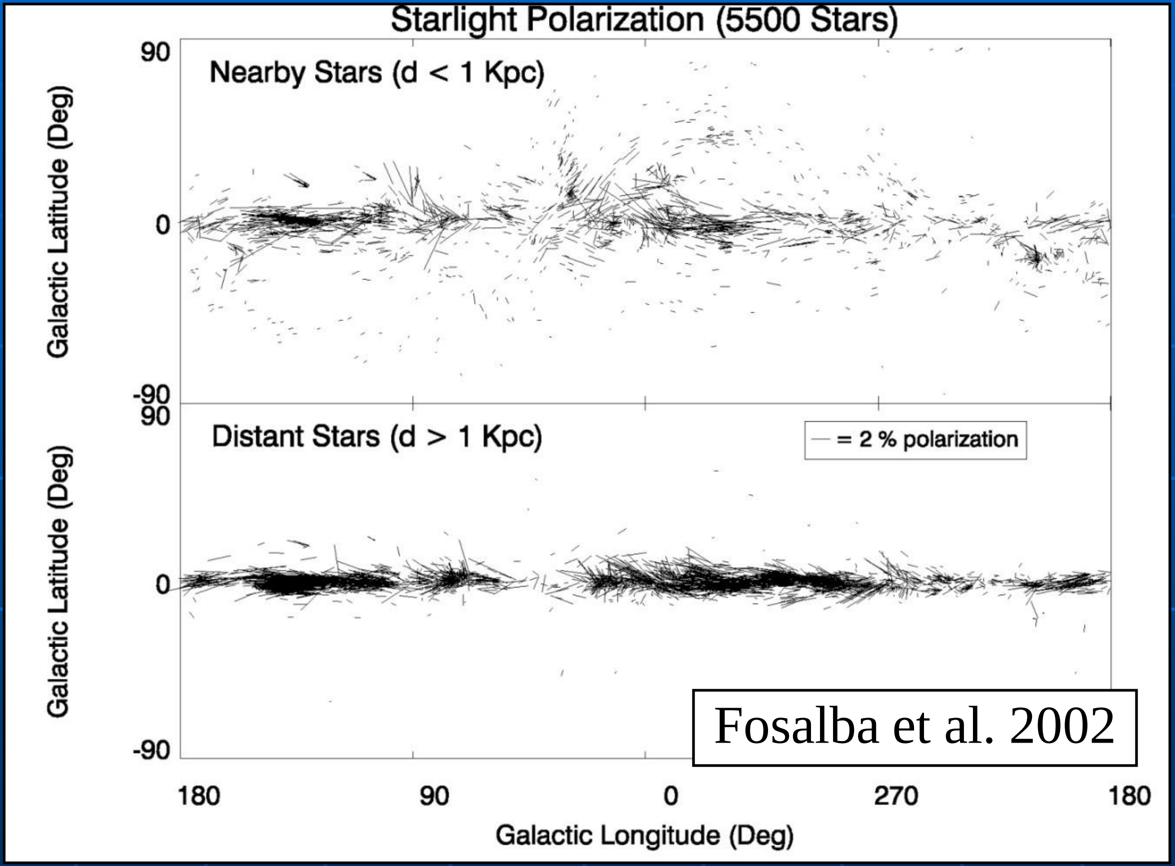


Average field strength ≈6 μG

Optical / IR /submm polarization
by dust grains aligned in magnetic fields

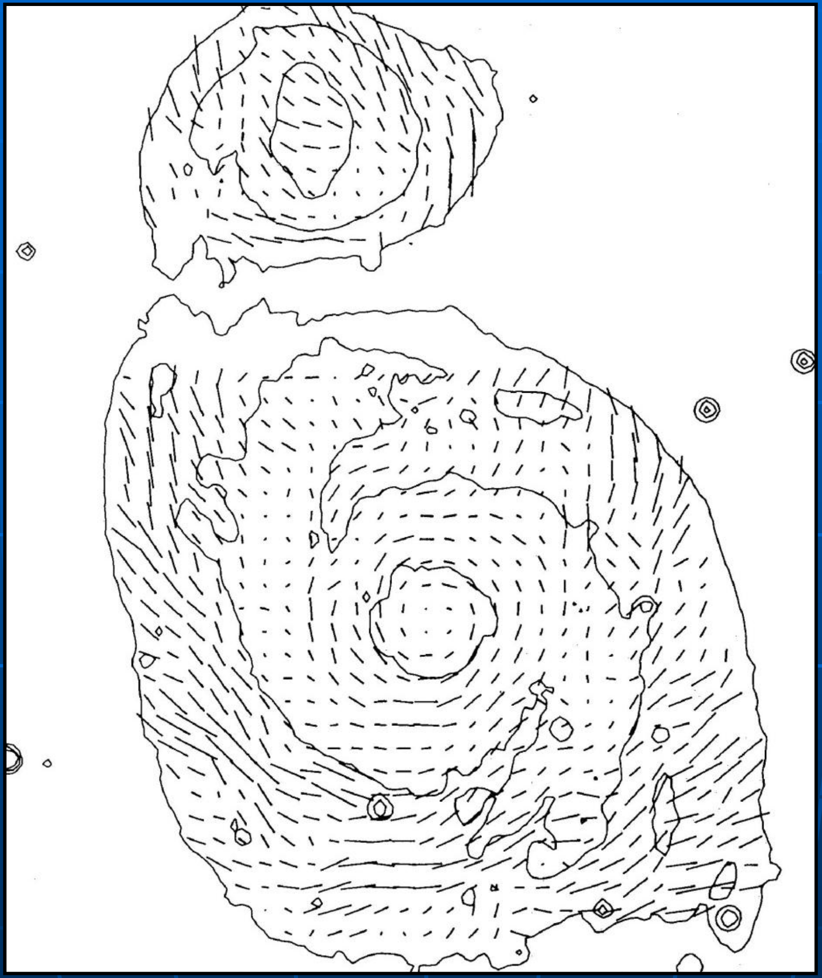


Starlight polarization: Ordered fields ($B_\perp$)



Large-scale ordered field along the plane

Starlight polarization in M 51

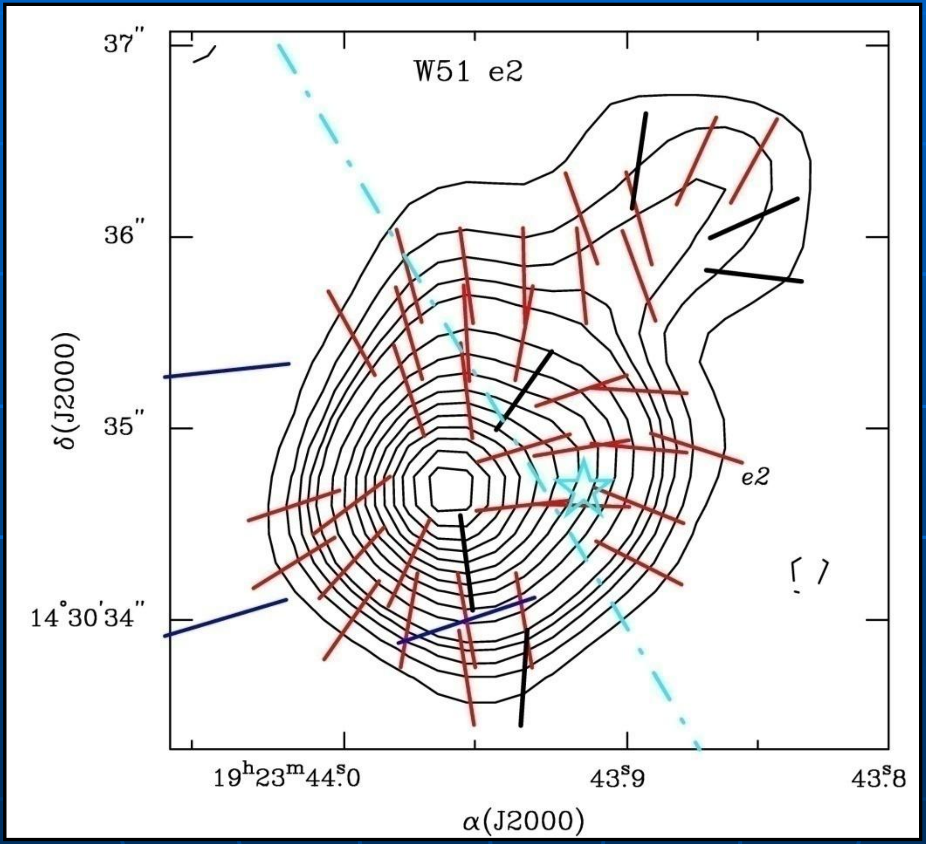


Scarrott et al. 1977

Large-scale spiral field

Submm polarization:
Ordered magnetic fields in a molecular gas disk

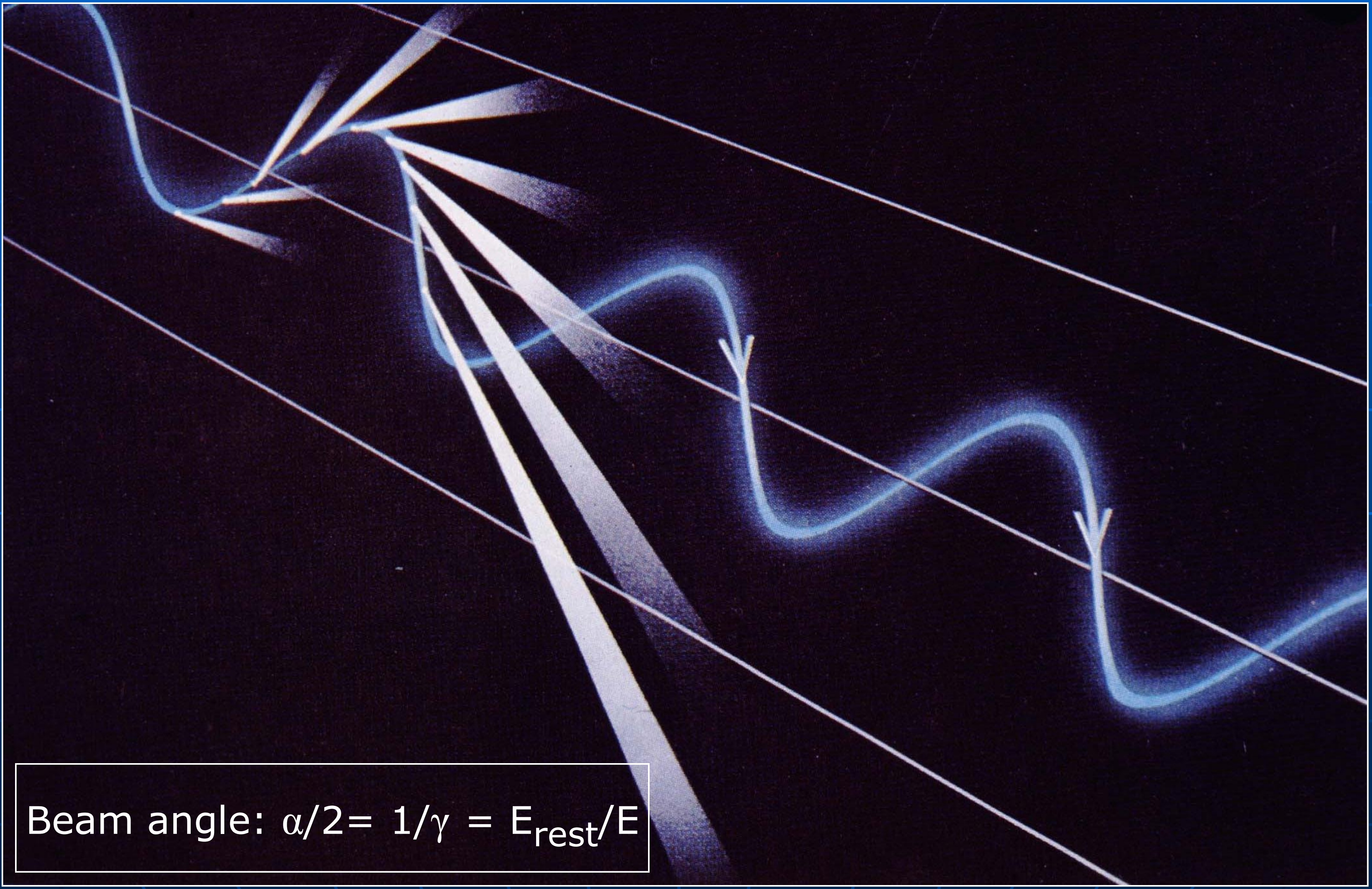
(SMA 870 μm, 0.02 pc resolution)



Tang et al. 2009

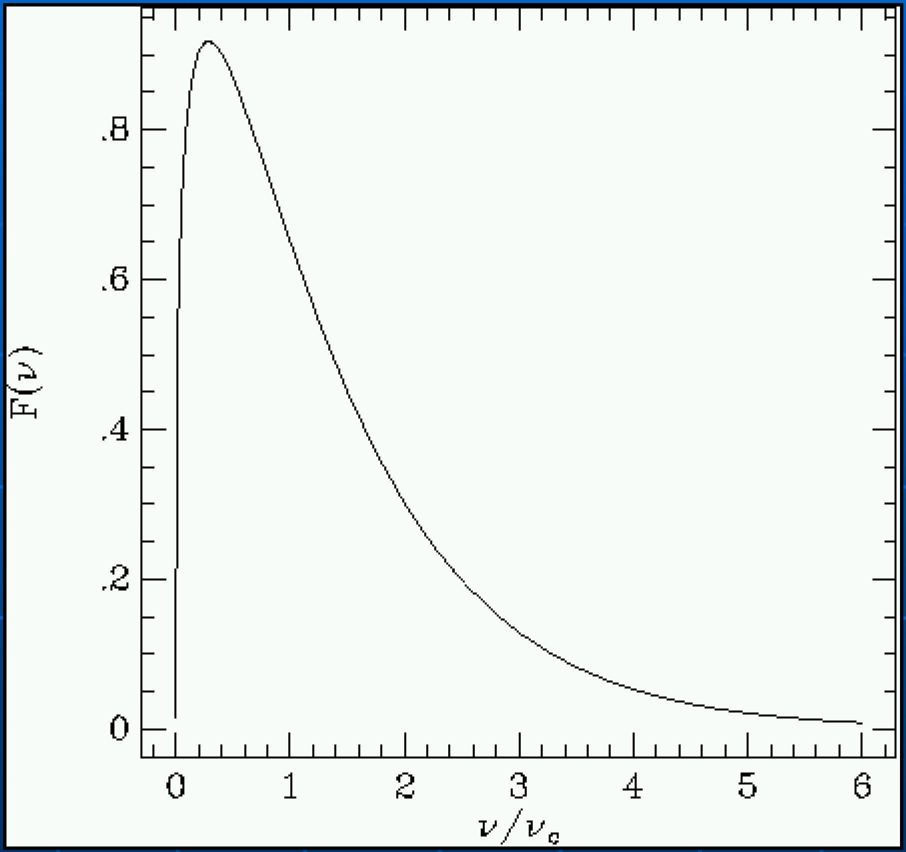
X-shaped field:
Ambipolar diffusion?

Synchrotron emission



Beam angle: $\alpha/2 = 1/\gamma = E_{rest}/E$

Synchrotron emission of a single electron



Critical frequency: $\nu_c = 3\,\gamma^2\, e\, B_\perp / (2\, m_e\, c)$

Synchrotron emission

- Ensemble of cosmic-ray electrons:

  Power-law energy spectrum with spectral index ε:

  $N(E)\, dE = N_0\, E^{-\varepsilon}\, dE$

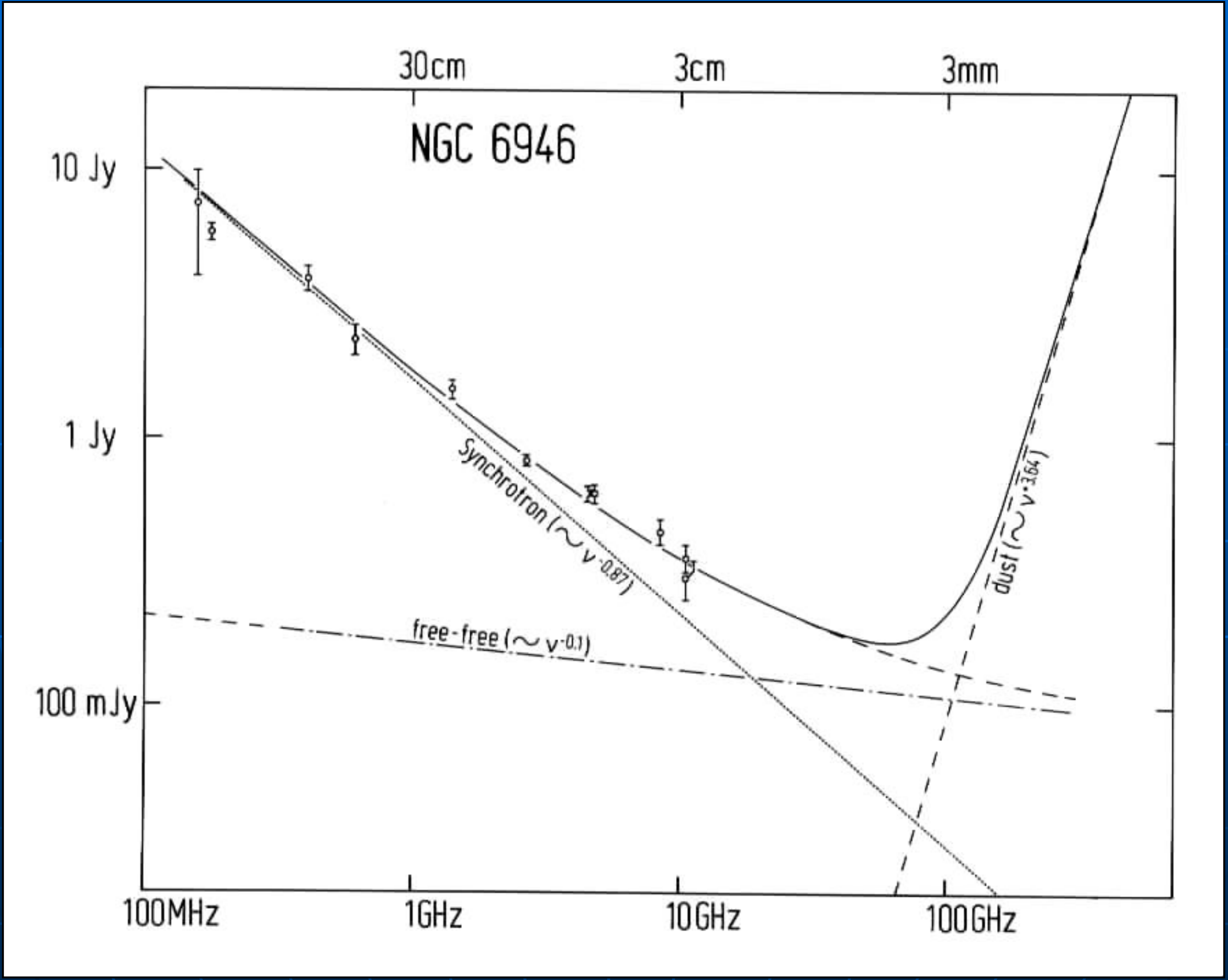
- Intensity of synchrotron spectrum:

  $I_\nu = c_5(\varepsilon) \int N_0\, B_\perp^{(\varepsilon+1)/2}\ (\nu/2c_1)^{-(\varepsilon-1)/2}\, dL$

- Synchrotron spectral index:

  $\alpha = (\varepsilon-1)/2$

  Strong shocks: $\varepsilon=2$, $\alpha=0.5$



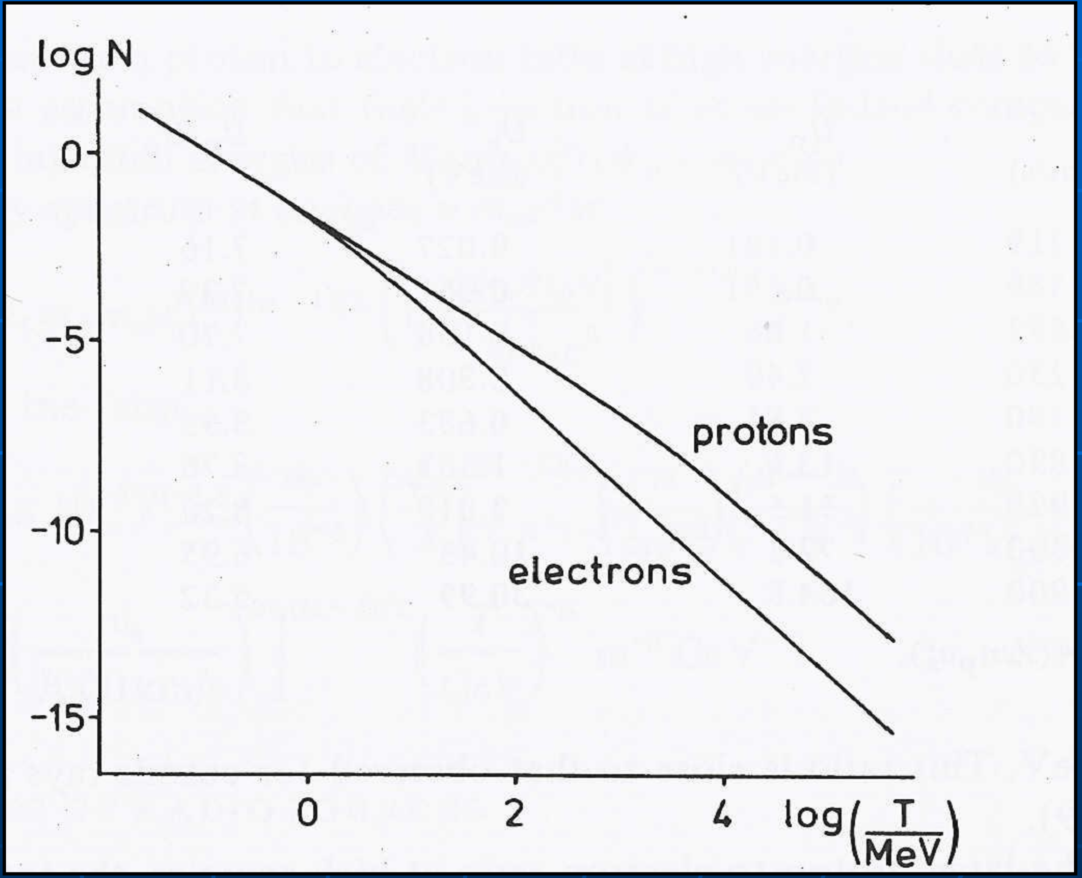
NGC 6946
30cm
3cm
3mm
10 Jy
1 Jy
100 mJy
100MHz
1GHz
10GHz
100GHz
Synchrotron (~ ν^-0.87)
free-free (~ ν^-0.1)
dust (~ ν^+3.64)

Energy spectra of cosmic rays

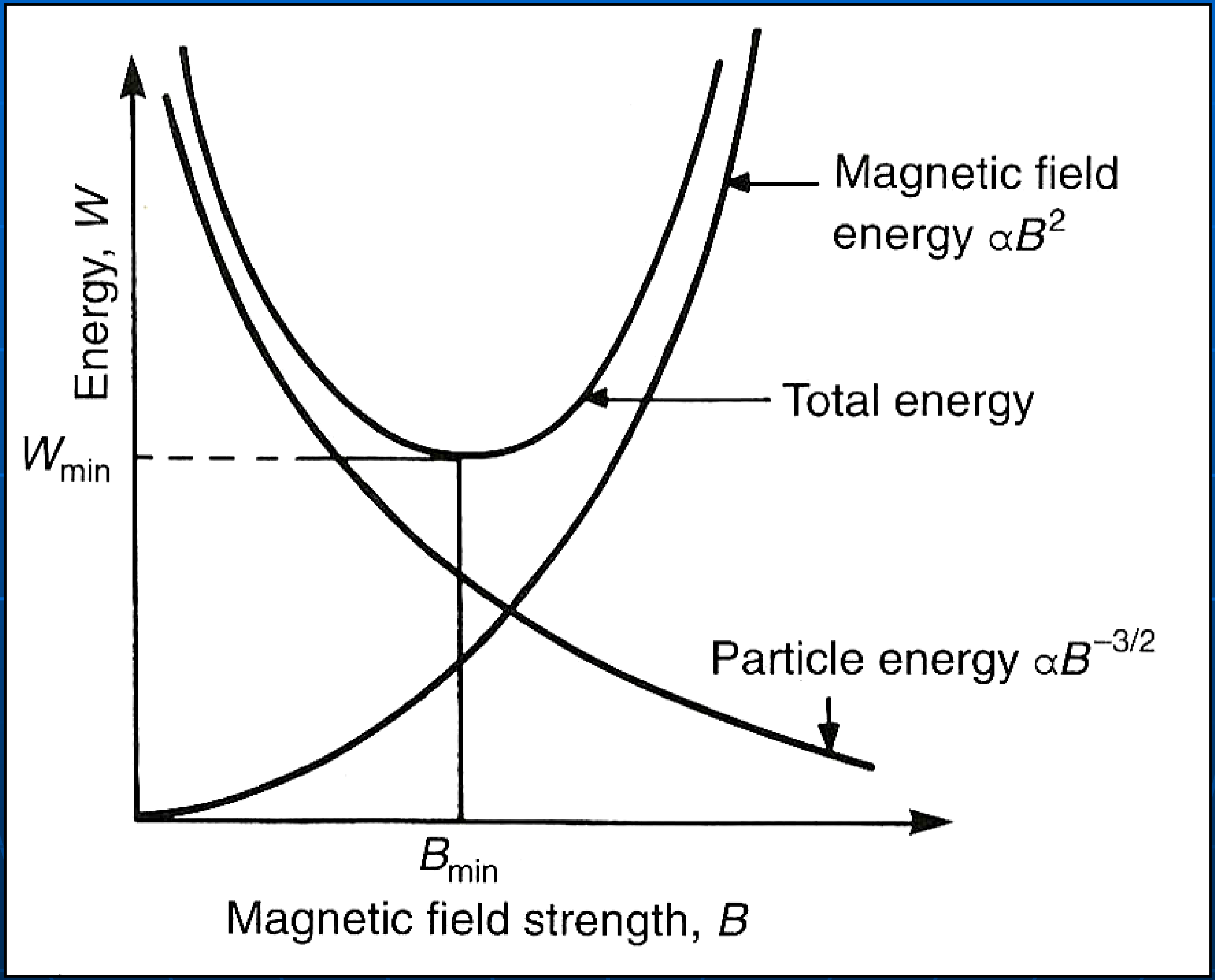
Bell 1978

Diffusive shock acceleration:
ε ≥ 2
(above rest energy)

E > 1 GeV:
K ≥ 100



Equipartition (minimum energy) field strengths



Longair, High Energy Astrophysics, 1994

Equipartition strength of the total field

(assuming equipartition between magnetic fields and cosmic rays)

Beck & Krause 2005

$$B_{eq,\perp} \propto \left(I_{sync} \, (K+1) \, / \, L \right)^{1/(3+\alpha)}$$

I_{sync}: Synchrotron intensity

L: Pathlength through source

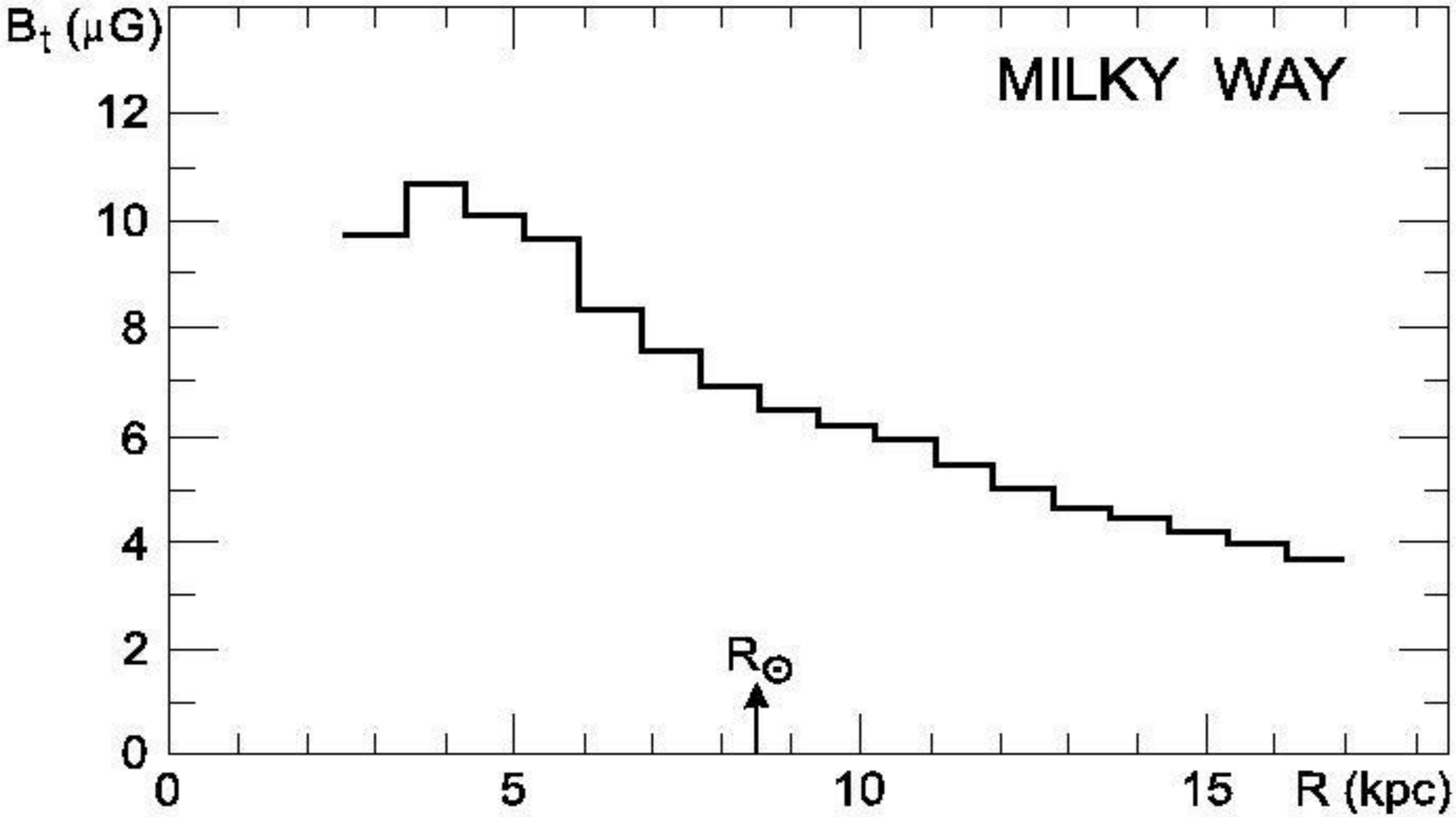
α: Synchrotron spectral index ($S \propto \nu^{-\alpha}$)

K: Ratio of cosmic-ray proton/electron number densities n_p/n_e

Usual assumption: K=100 (no energy losses of CR electrons)

Equipartition field in the Milky Way

(Berkhuijsen, in Wielebinski & Beck 2005)

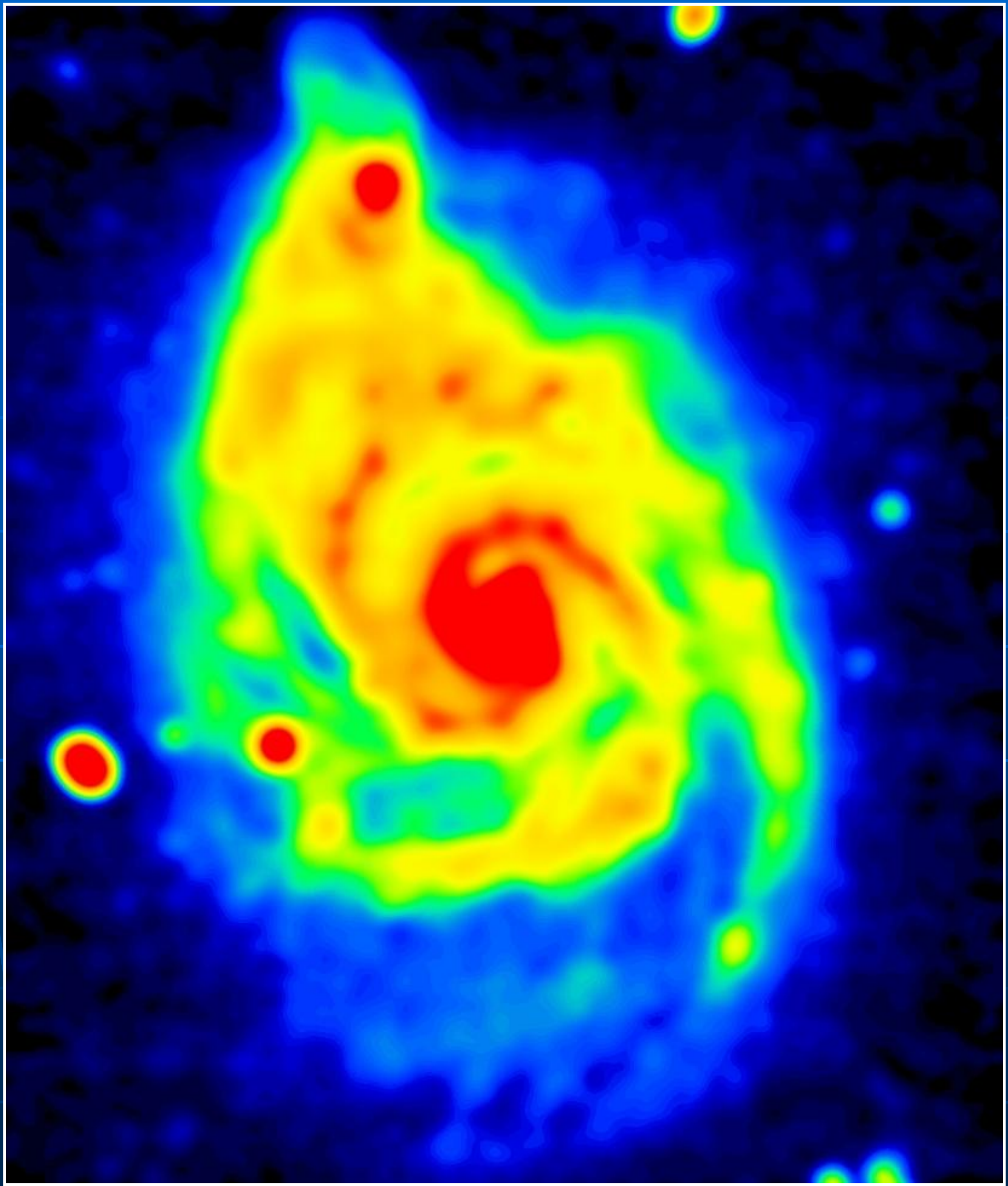


Consistent with estimates from γ rays

(Strong et al. 2000)

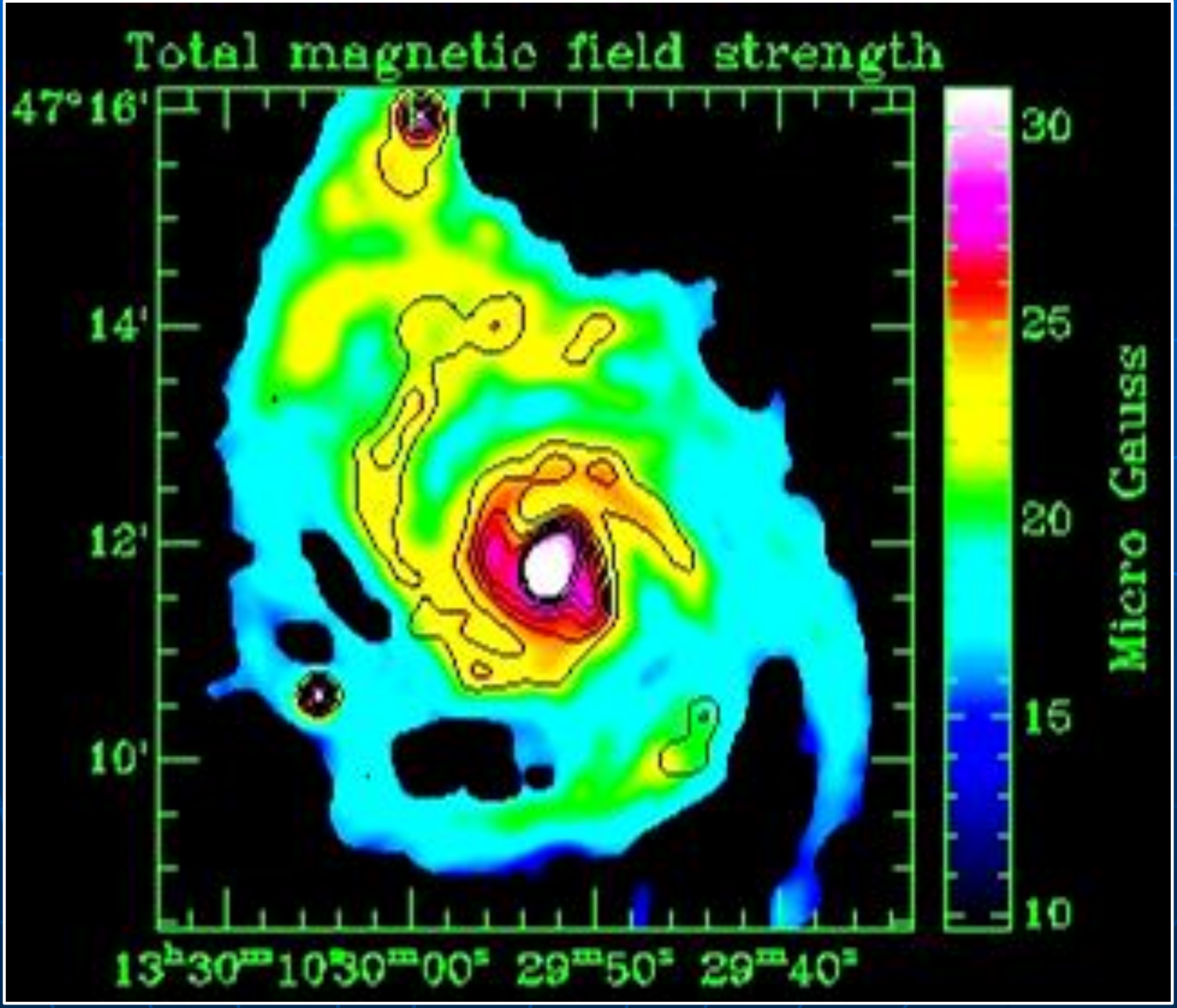
M 51

20cm VLA
Total intensity
(Fletcher et al. 2010)



Equipartition field strengths in M 51

Fletcher et al. 2010



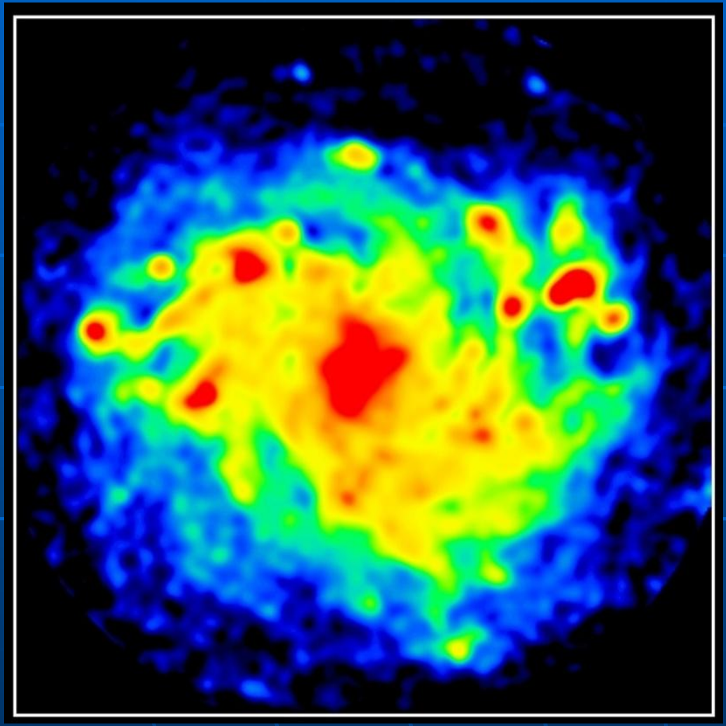
Magnetic field strengths in spiral galaxies

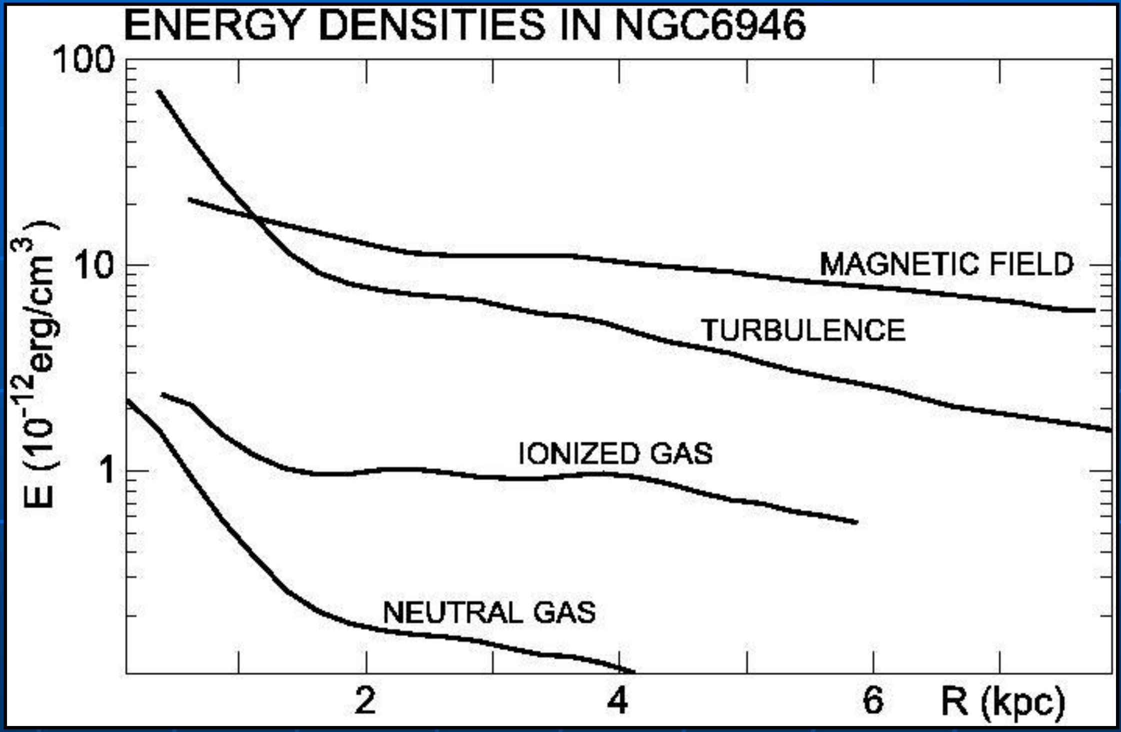
(from synchrotron intensity, assuming energy equipartition with cosmic rays)

Total field in spiral arms:	**20 - 30 μG**
Regular field in interarm regions:	**5 - 15 μG**
Total field in circum-nuclear rings:	**40 – 100 μG**
Total field in galaxy center filaments:	**≈ 1 mG**

Magnetic energy density
(assuming equipartition with cosmic rays)

Beck 2007

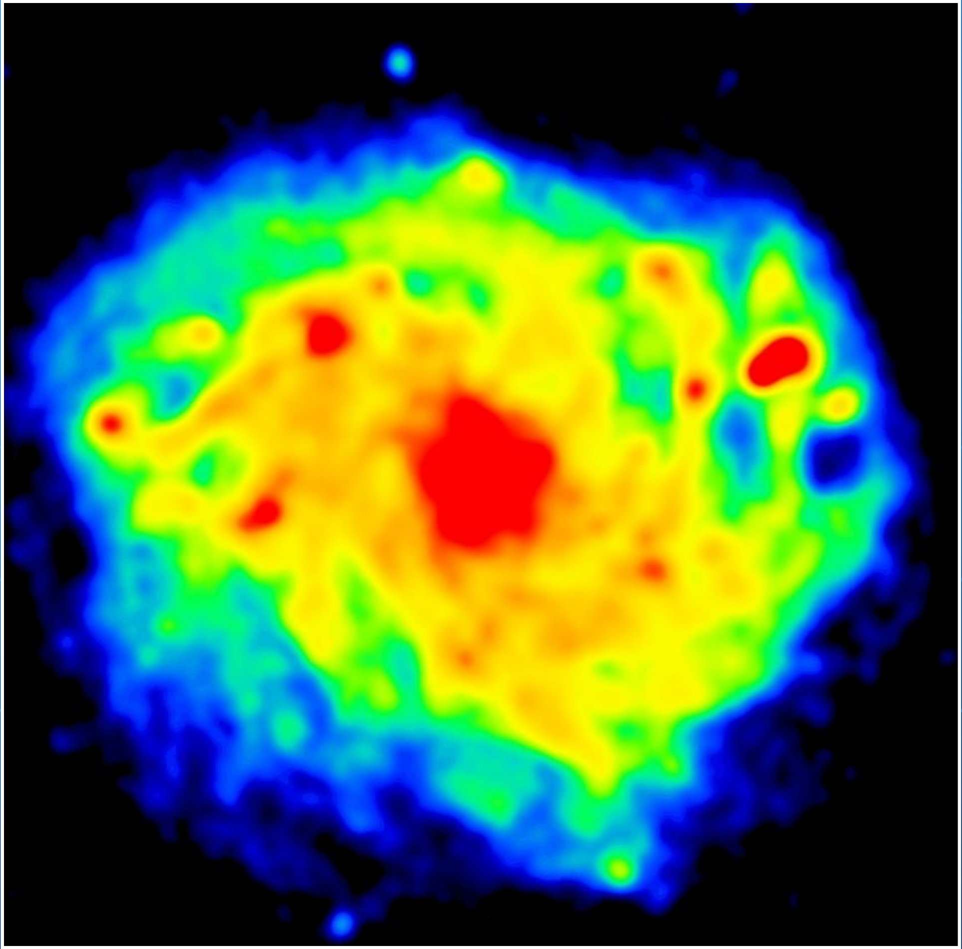




- Magnetic energy density is similar to that of turbulence
- ISM is a low-beta plasma
- No modelling without magnetic fields !

NGC 6946

20cm VLA
Total intensity
(Beck 2007)



Radio synchrotron disk:

Extent is limited by the energy losses of the cosmic-ray electrons (few GeV)

Linear polarization of synchrotron emission

B-vector: oriented along the magnetic field line
(if no Faraday rotation occurs)

Intrinsic degree of linear polarization
(if no depolarization occurs):

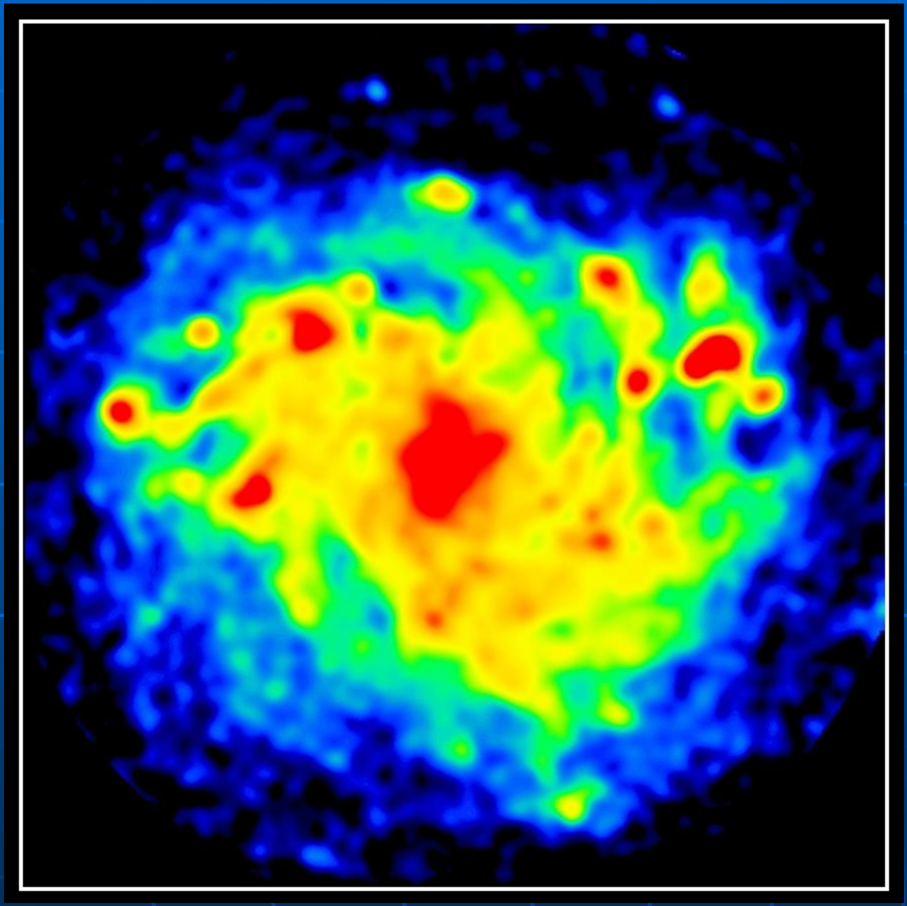
$$p_o = (\varepsilon+1) / (\varepsilon+7/3)$$
$$= (\alpha+1) / (\alpha+5/3)$$

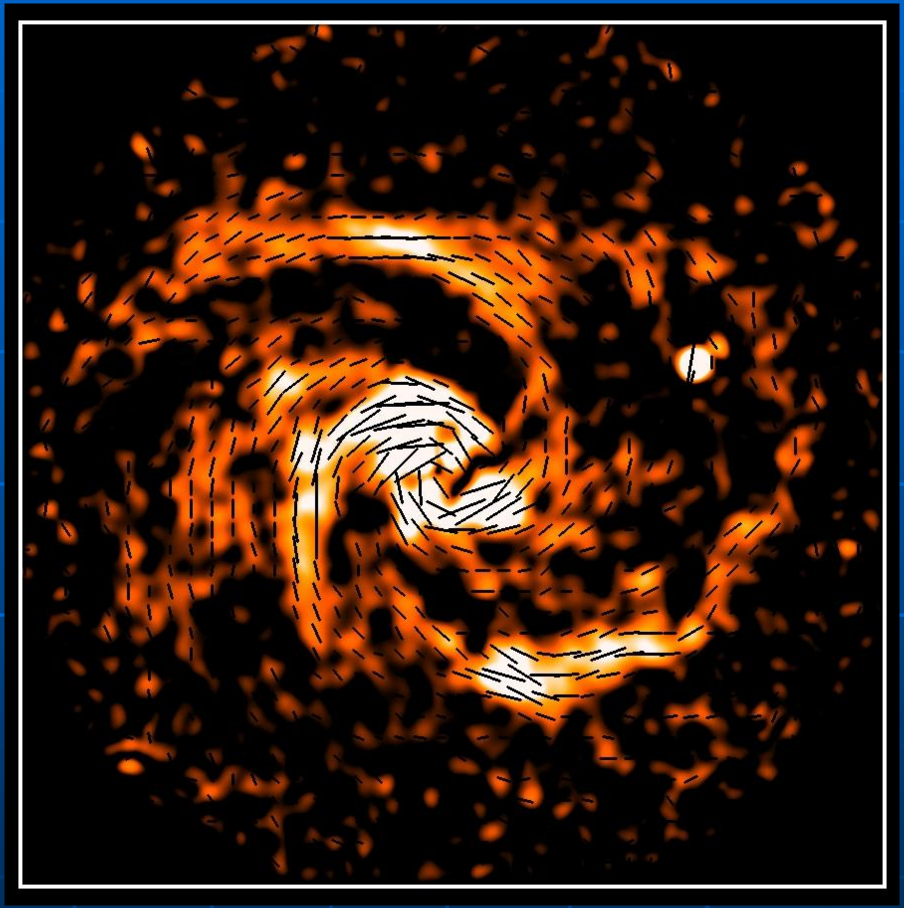
Typical value: $\alpha=0.9$, $p_o=74\%$

Circular polarization: generally negligible

Synchrotron polarization

Beck & Hoernes 1996



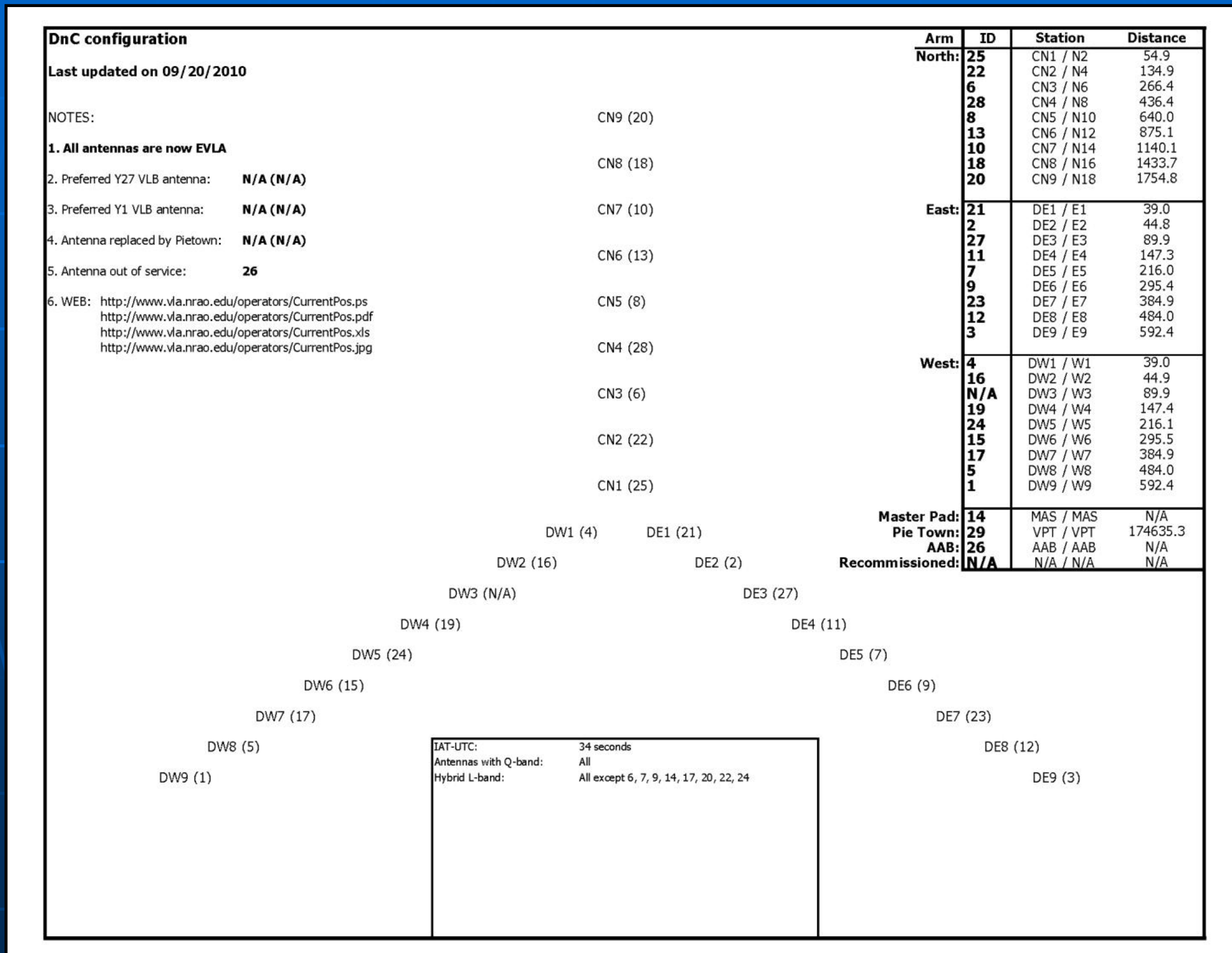


NGC 6946 Total and polarized intensity at 6cm

Combination of interferometer and single-dish data

VLA D-array

VLA: current configuration (DnC)



DnC configuration

Last updated on 09/20/2010

NOTES:

1. All antennas are now EVLA

2. Preferred Y27 VLB antenna: **N/A (N/A)**

3. Preferred Y1 VLB antenna: **N/A (N/A)**

4. Antenna replaced by Pietown: **N/A (N/A)**

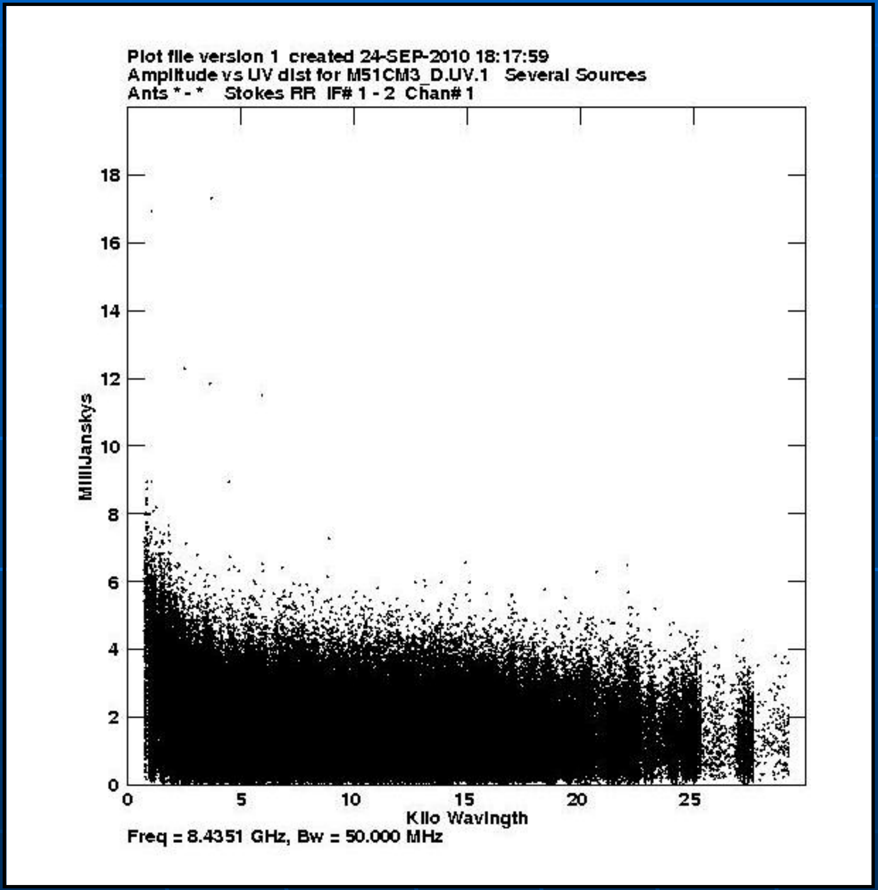
5. Antenna out of service: **26**

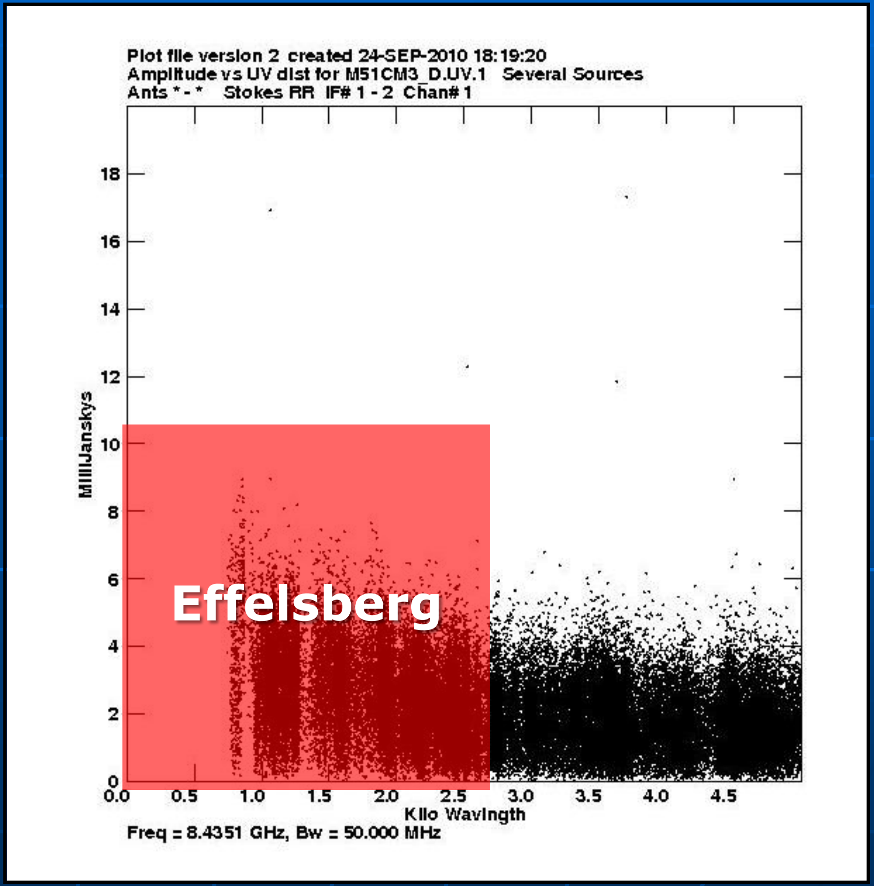
6. WEB: http://www.vla.nrao.edu/operators/CurrentPos.ps
http://www.vla.nrao.edu/operators/CurrentPos.pdf
http://www.vla.nrao.edu/operators/CurrentPos.xls
http://www.vla.nrao.edu/operators/CurrentPos.jpg

Arm	ID	Station	Distance
North:	**25**	CN1 / N2	54.9
	22	CN2 / N4	134.9
	6	CN3 / N6	266.4
	28	CN4 / N8	436.4
	8	CN5 / N10	640.0
	13	CN6 / N12	875.1
	10	CN7 / N14	1140.1
	18	CN8 / N16	1433.7
	20	CN9 / N18	1754.8
East:	**21**	DE1 / E1	39.0
	2	DE2 / E2	44.8
	27	DE3 / E3	89.9
	11	DE4 / E4	147.3
	7	DE5 / E5	216.0
	9	DE6 / E6	295.4
	23	DE7 / E7	384.9
	12	DE8 / E8	484.0
	3	DE9 / E9	592.4
West:	**4**	DW1 / W1	39.0
	16	DW2 / W2	44.9
	N/A	DW3 / W3	89.9
	19	DW4 / W4	147.4
	24	DW5 / W5	216.1
	15	DW6 / W6	295.5
	17	DW7 / W7	384.9
	5	DW8 / W8	484.0
	1	DW9 / W9	592.4
Master Pad:	**14**	MAS / MAS	N/A
Pie Town:	**29**	VPT / VPT	174635.3
AAB:	**26**	AAB / AAB	N/A
Recommissioned:	**N/A**	N/A / N/A	N/A

IAT-UTC:	34 seconds
Antennas with Q-band:	All
Hybrid L-band:	All except 6, 7, 9, 14, 17, 20, 22, 24

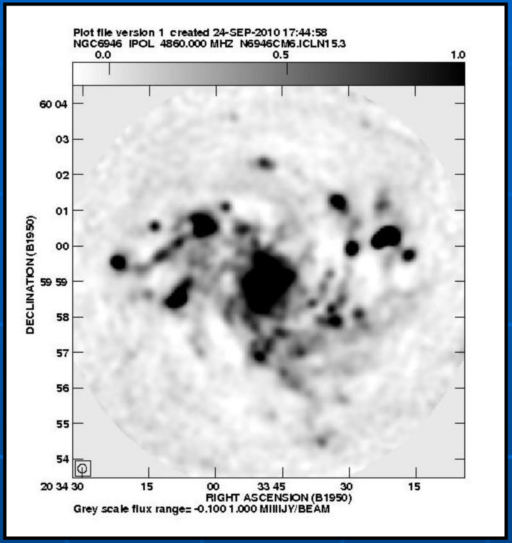
(u,v) coverage of VLA data
(M51 3cm D array)

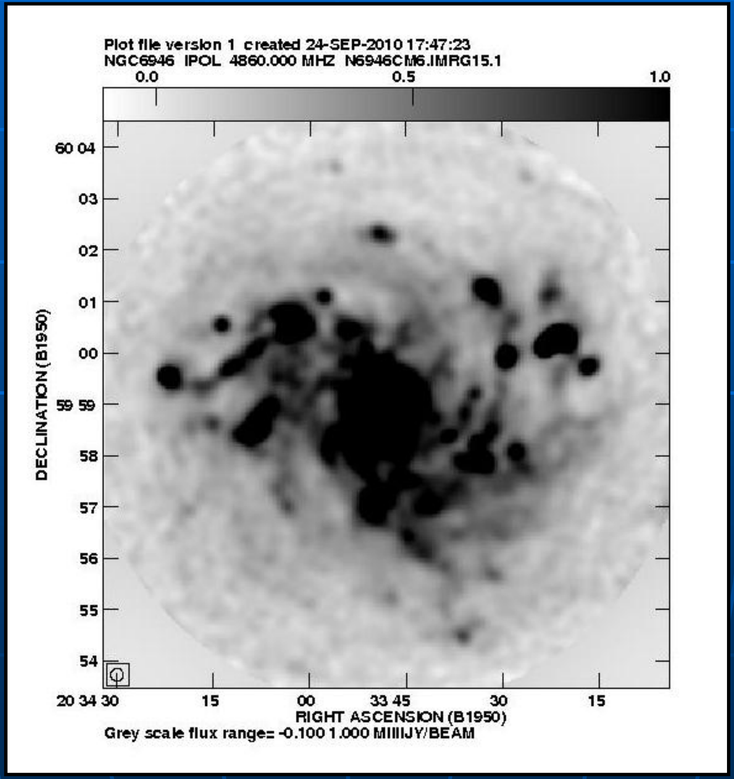


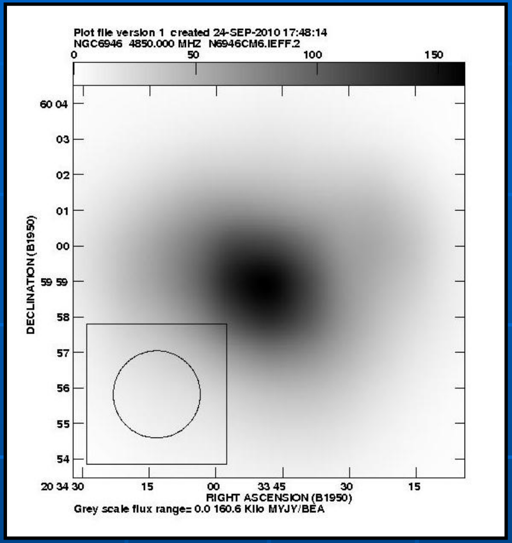


Minimum baseline: 25m ≈ 700 kλ

Combination of VLA and Effelsberg data
(NGC6946 6cm Stokes I)

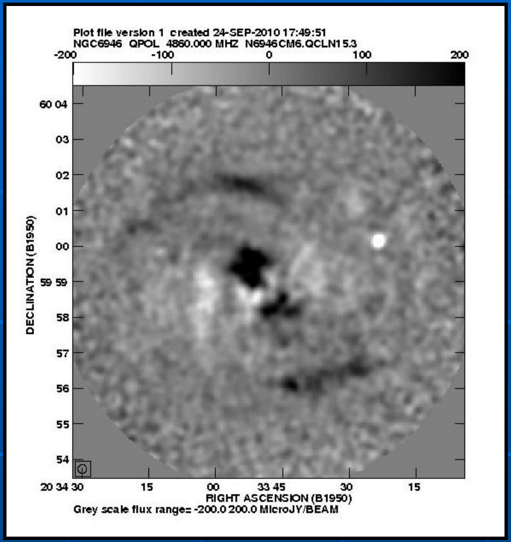


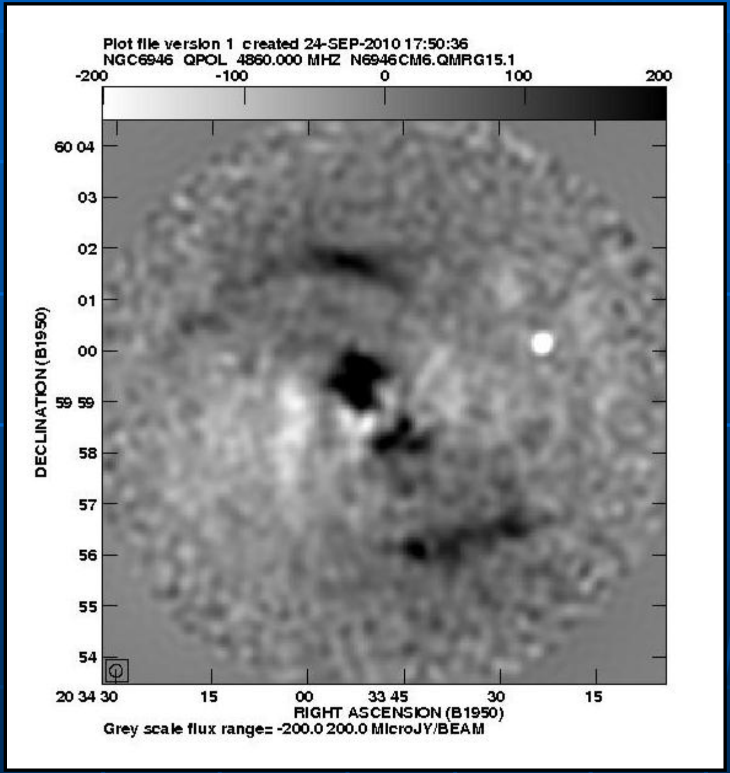


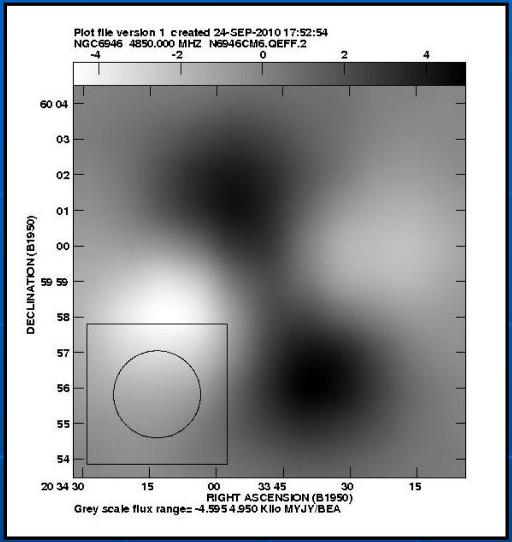


VLA alone: ≈ 50% total flux

Combination of VLA and Effelsberg data
(NGC6946 6cm Stokes Q)



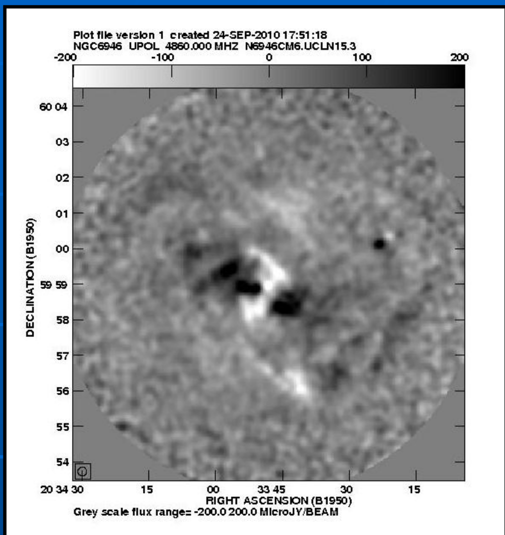


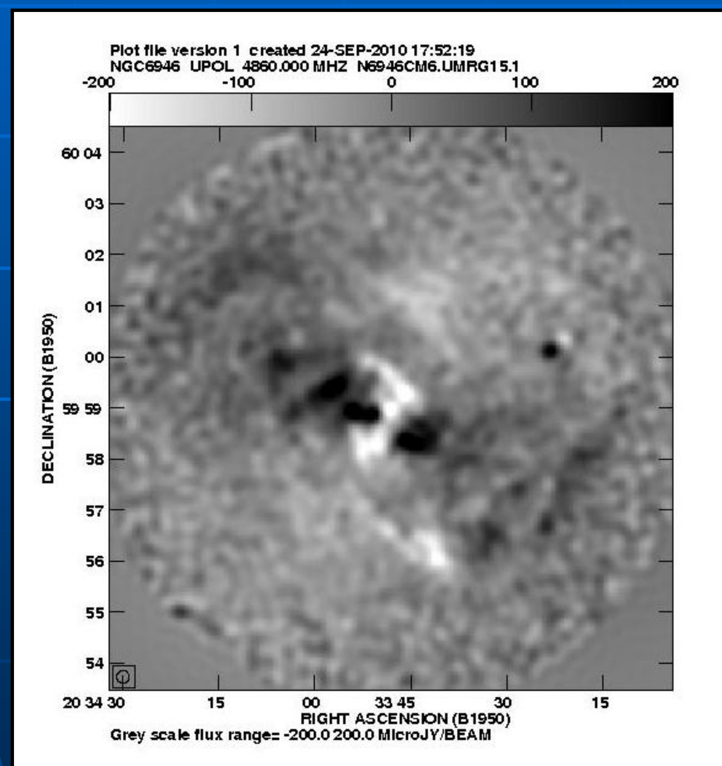


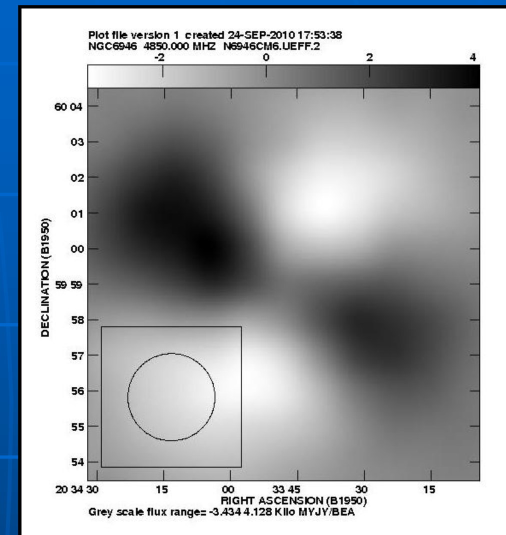
Missing large-scale structure: Wrong polarization angle !

(see talk by Wolfgang Reich)

Combination of VLA and Effelsberg data
(NGC6946 6cm Stokes U)





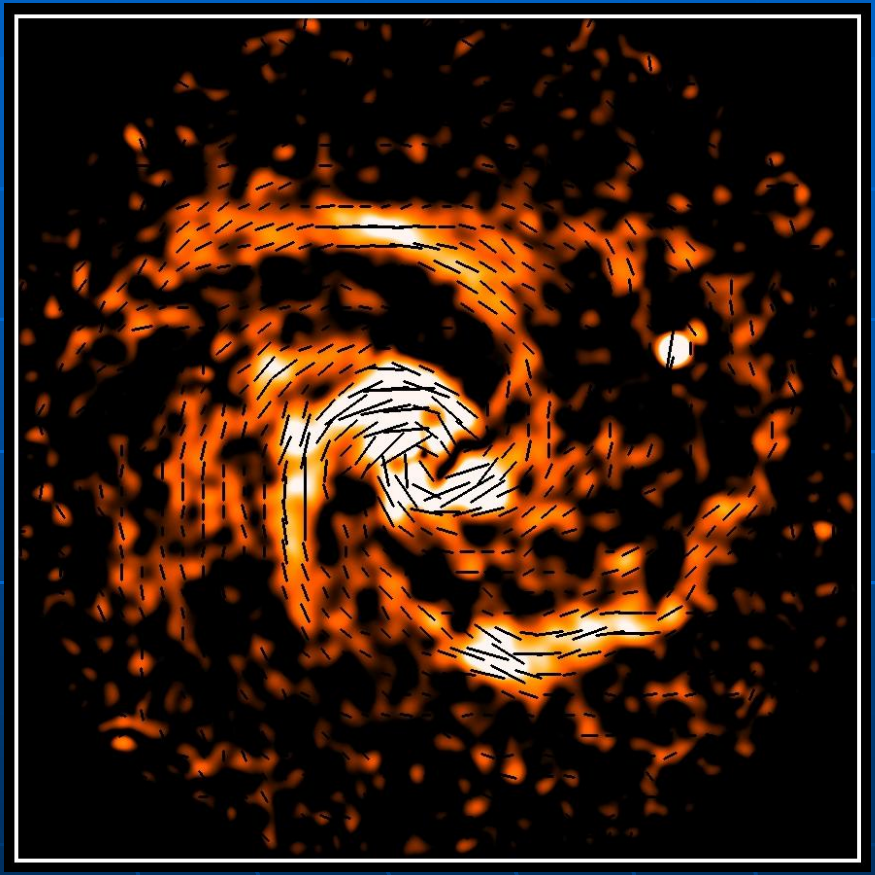


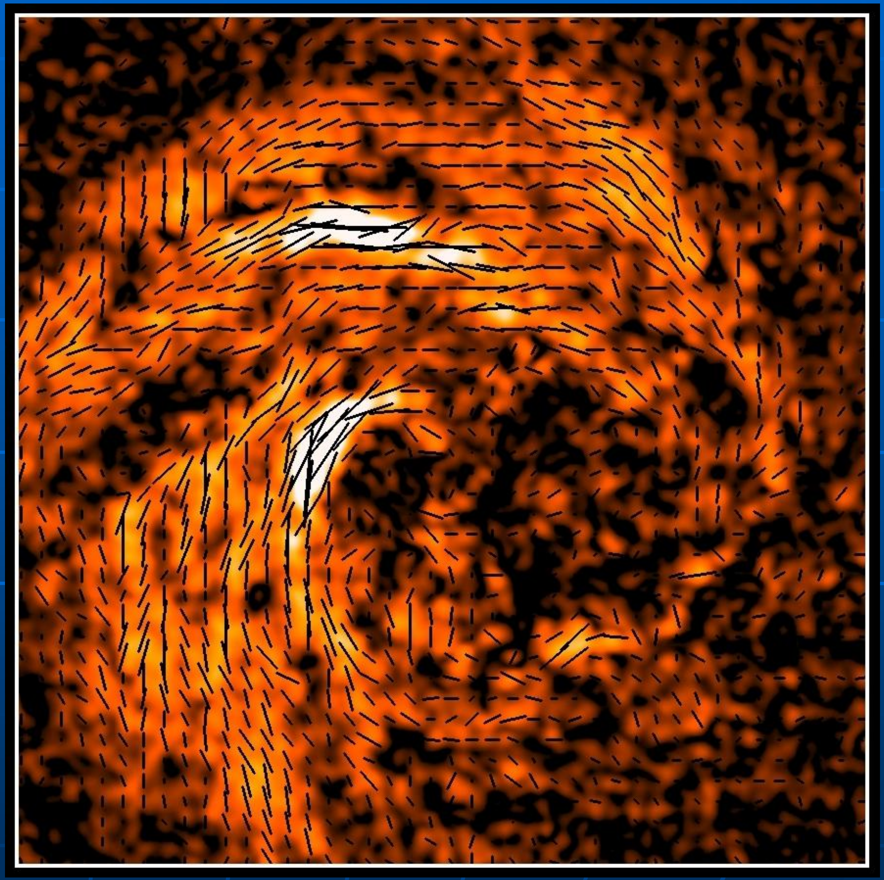
Missing large-scale structure: Wrong polarization angle !

Single dish observations
are crucial
for any synthesis telescope
to fill the missing spacings
and to obtain
accurate polarization angles

Faraday depolarization

Beck 2007





NGC 6946 Polarized intensity at 6cm and 20.5cm

Degree of polarization:

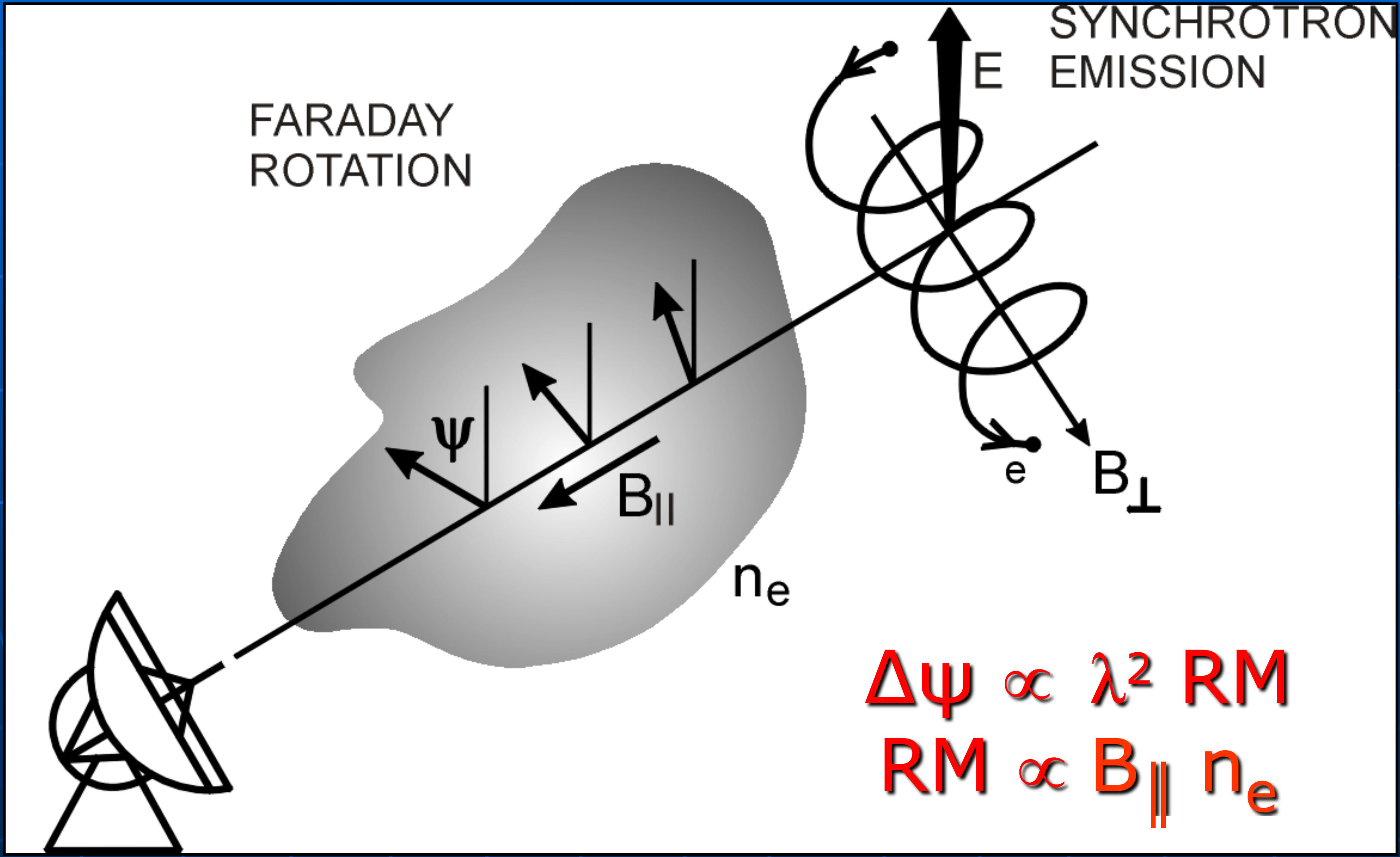
≤ 5% in spiral arms,

20 - 60% in magnetic arms at ≥5 GHz,

≤10% everywhere at ≤1.4 GHz

(Faraday depolarization)

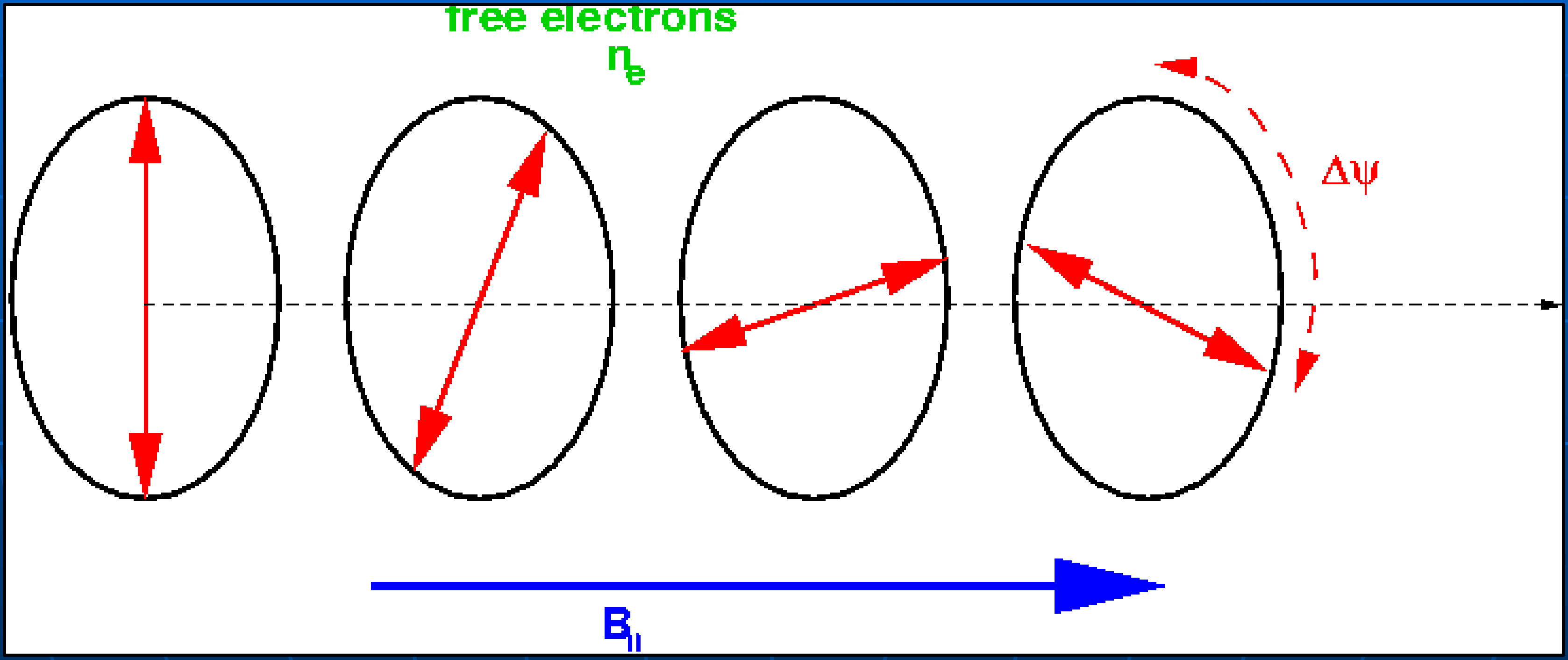
Faraday rotation



$$\Delta\psi \propto \lambda^2 \, RM$$

$$RM \propto B_{\parallel} \, n_e$$

Faraday rotation



Faraday rotation

$$\Delta\chi = 0.81 \text{ (rad) } \lambda \text{ (m)}^2 \int n_e \text{ (cm}^{-3}\text{) } B_{reg\,\|} \text{ (}\mu\text{G) } dl \text{ (pc)}$$

$$= \mathbf{RM}\ \lambda \text{ (m)}^2$$

Classical measurement of RM: Fitting $\Delta\chi$ as a function of λ^2

Typical Faraday rotation measures

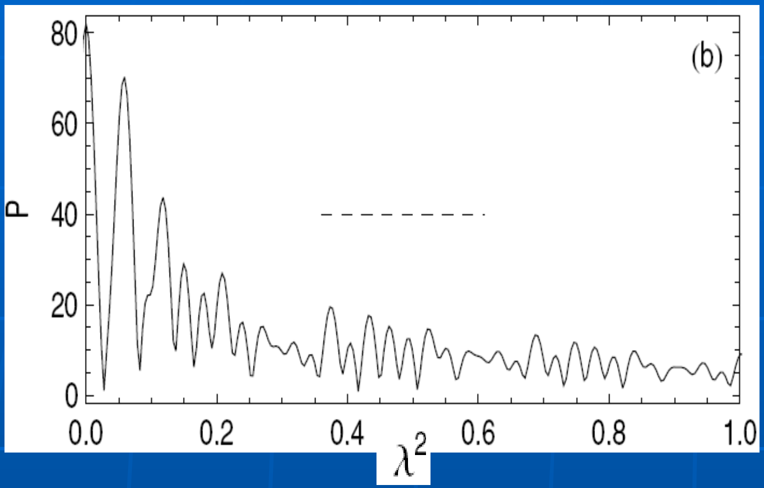
- Intergalactic medium: ≈ 0.01-0.1 rad m^{-2}
- Galactic halos: ≈ 0.1-1 rad m^{-2}
- Galactic disks: ≈ 10-100 rad m^{-2}
- Galaxy clusters: ≈ 100-1000 rad m^{-2}
- Cooling cores of clusters: ≈ 1000-10000 rad m^{-2}

Faraday rotation angles

|RM|	=	100	10	1	0.1 rad m^{-2}
1400 MHz	$\Delta\chi$ =	263°	26°	3°	0.3°
200 MHz	$\Delta\chi$ =	12900°	1290°	129°	13°
120 MHz	$\Delta\chi$ =	35800°	3580°	358°	36°

3-D images
need
polarization

Spectro-polarimetry in radio continuum



$F(\phi)$?

What is the source distribution along the line of sight (in “Faraday space”) ?

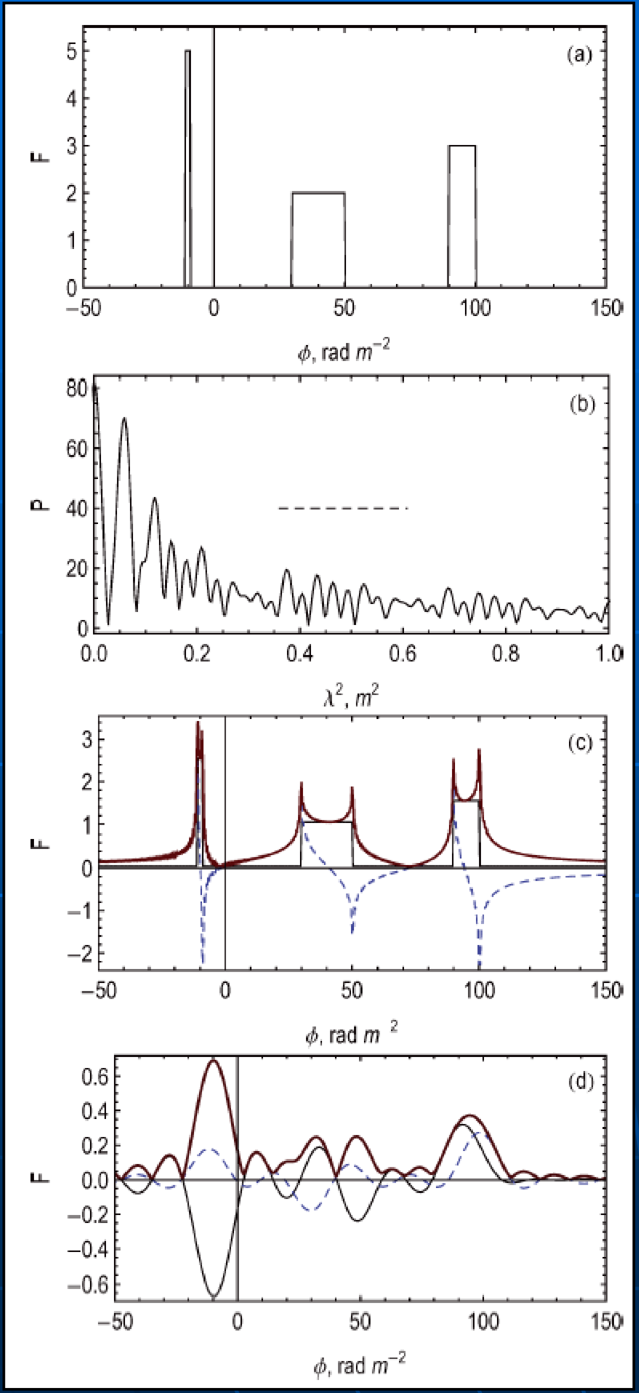
Burn (1966) noted that the observed complex polarized intensity *P* is related to the Faraday spectrum *F*(*φ*) as:

$$P(\lambda^2) = \int_{-\infty}^{\infty} F(\phi) e^{2i\phi\lambda^2} \, d\phi$$

$$F(\phi) = \frac{1}{\pi} \hat{P}(k),$$

$$k = 2\phi$$

Fourier transform ("**RM Synthesis**") first introduced into multi-channel polarization observations by Brentjens & de Bruyn (2005)



(a) Structures in Faraday space (real part)

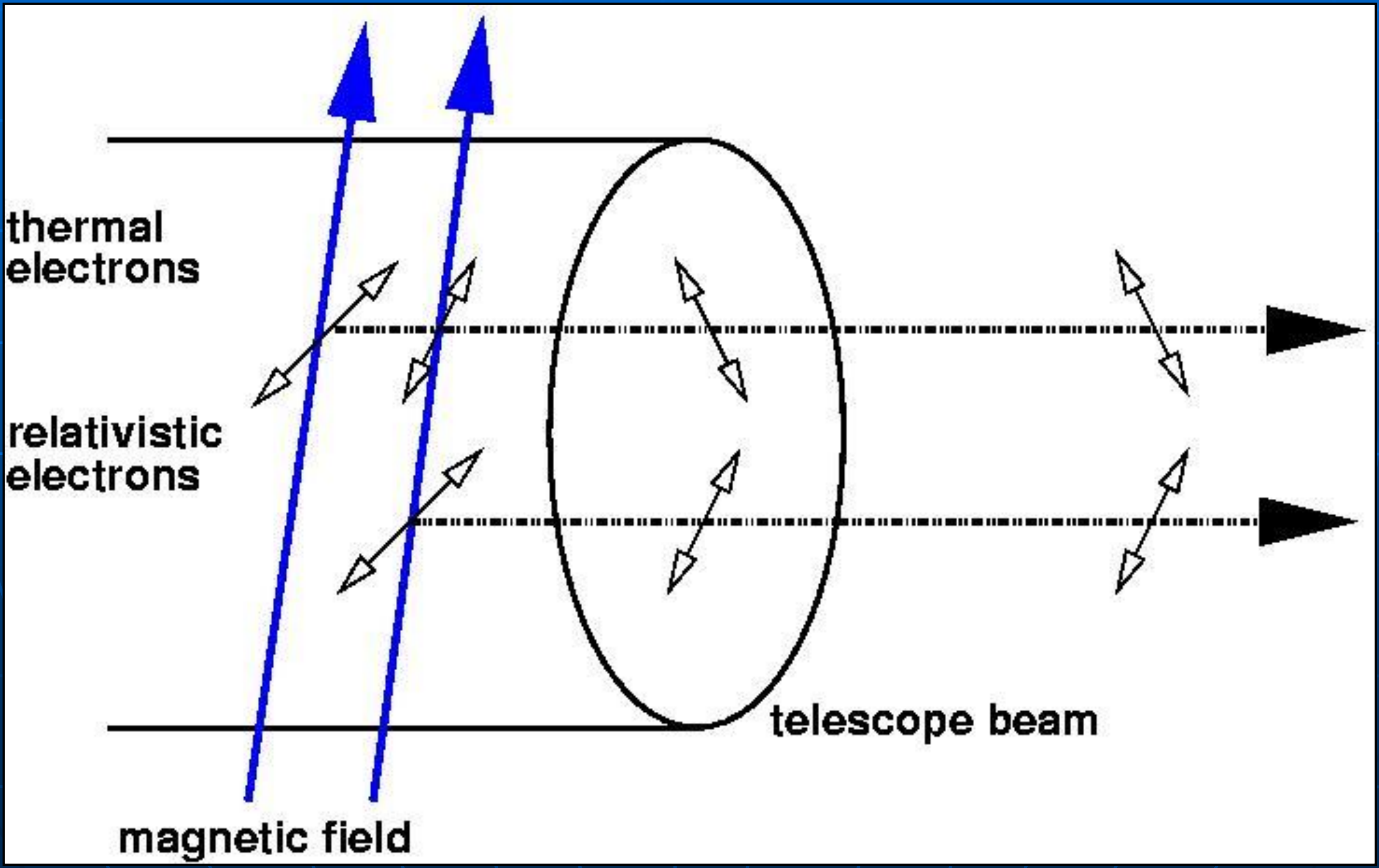
(b) Observed polarized intensity

(c) RM Synthesis (real and imaginary part) for full coverage in wavelength

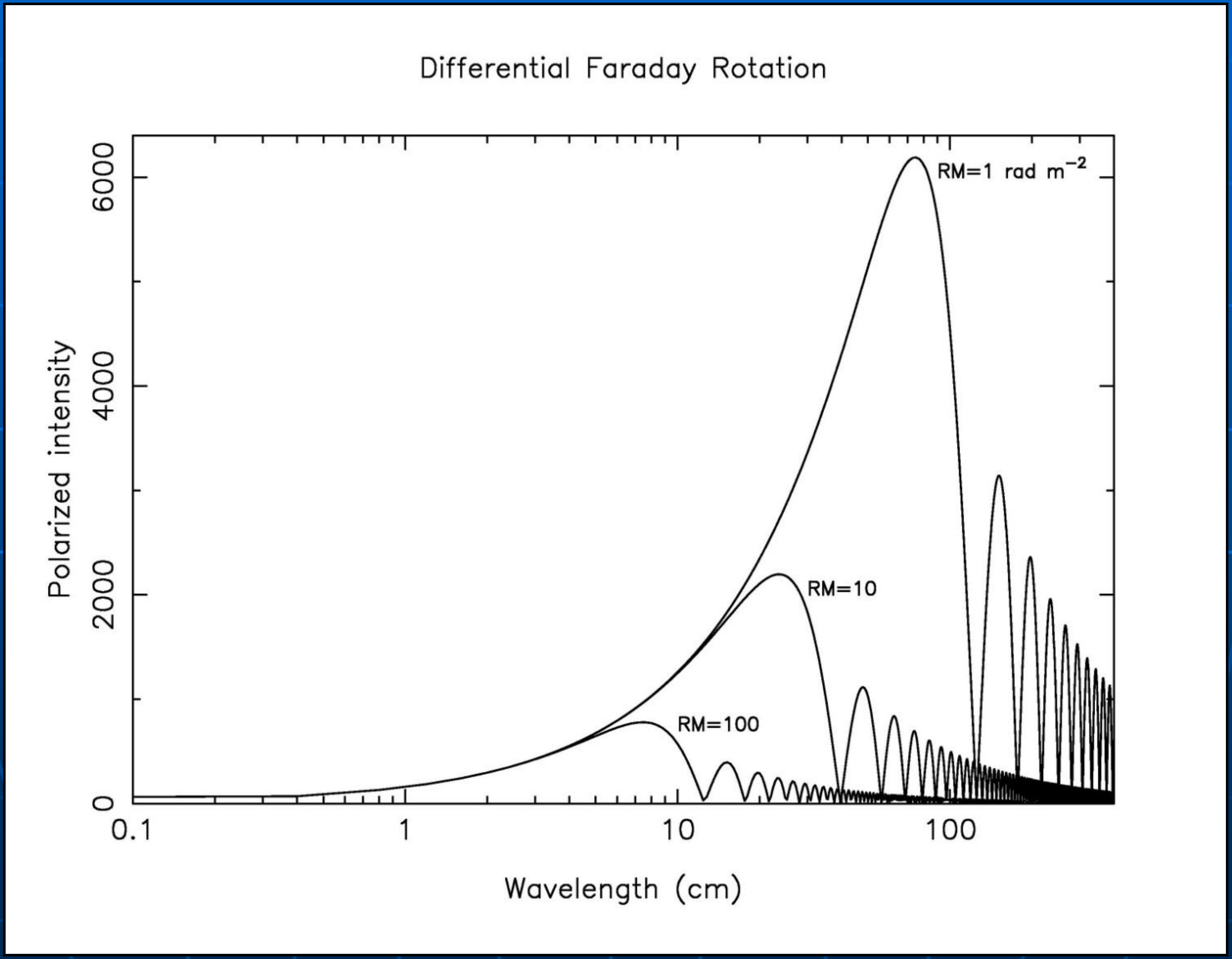
(d) RM Synthesis for a limited wavelength coverage (0.6-0.8m)

Differential Faraday rotation
(wavelength-dependent)

Fletcher et al. 2004

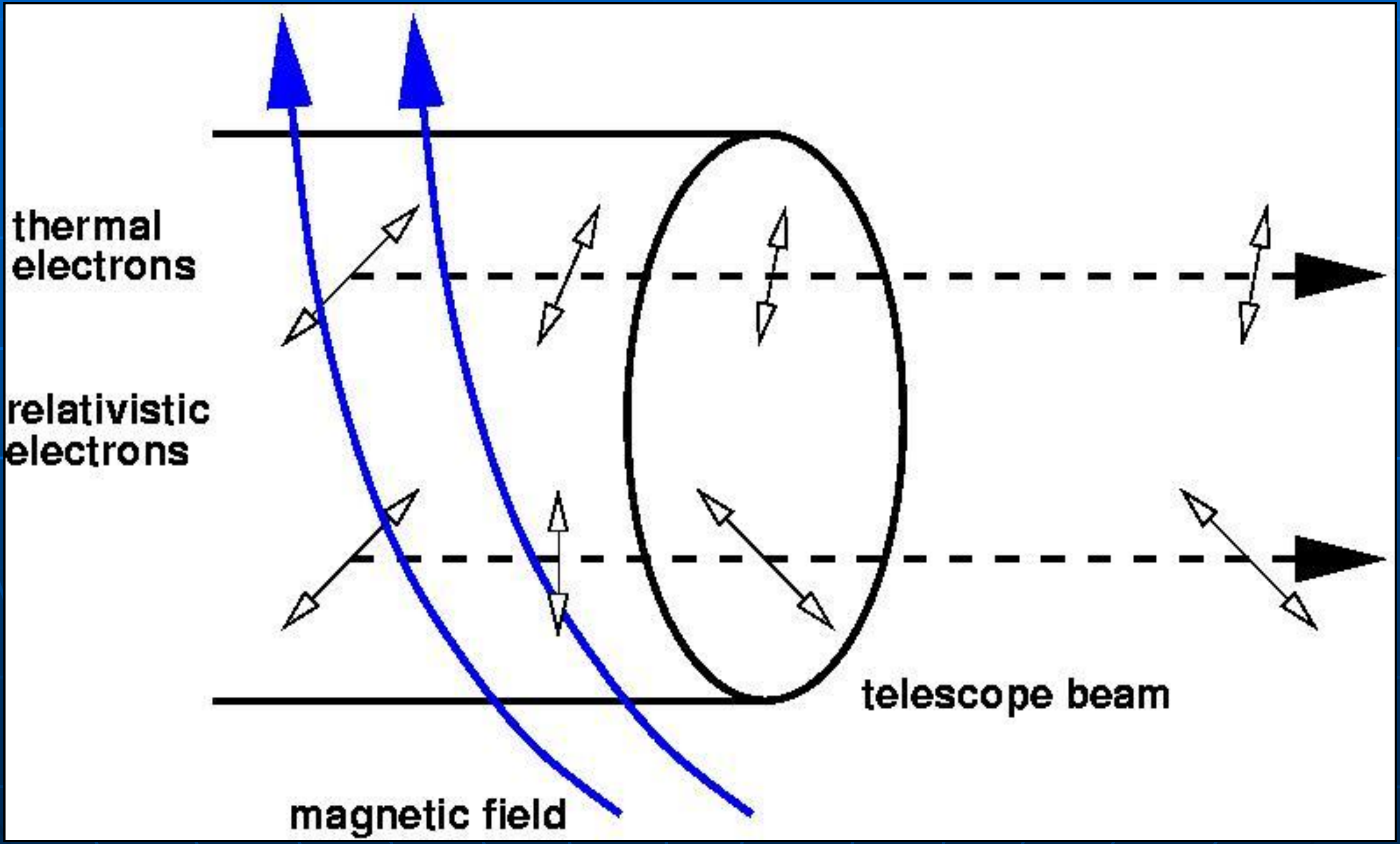


Maximum polarization



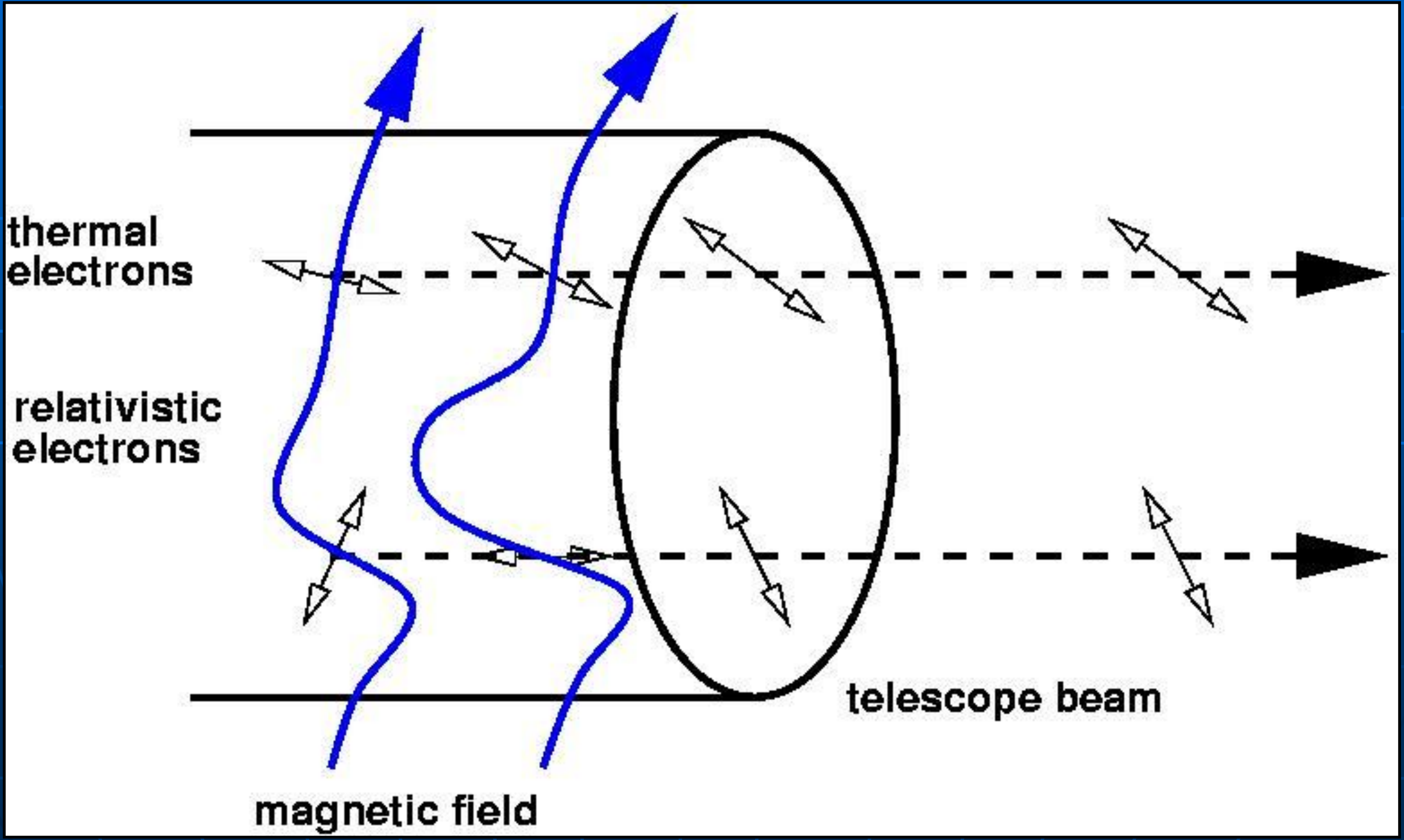
Rotation measure gradient
(wavelength-dependent)

Fletcher et al. 2004

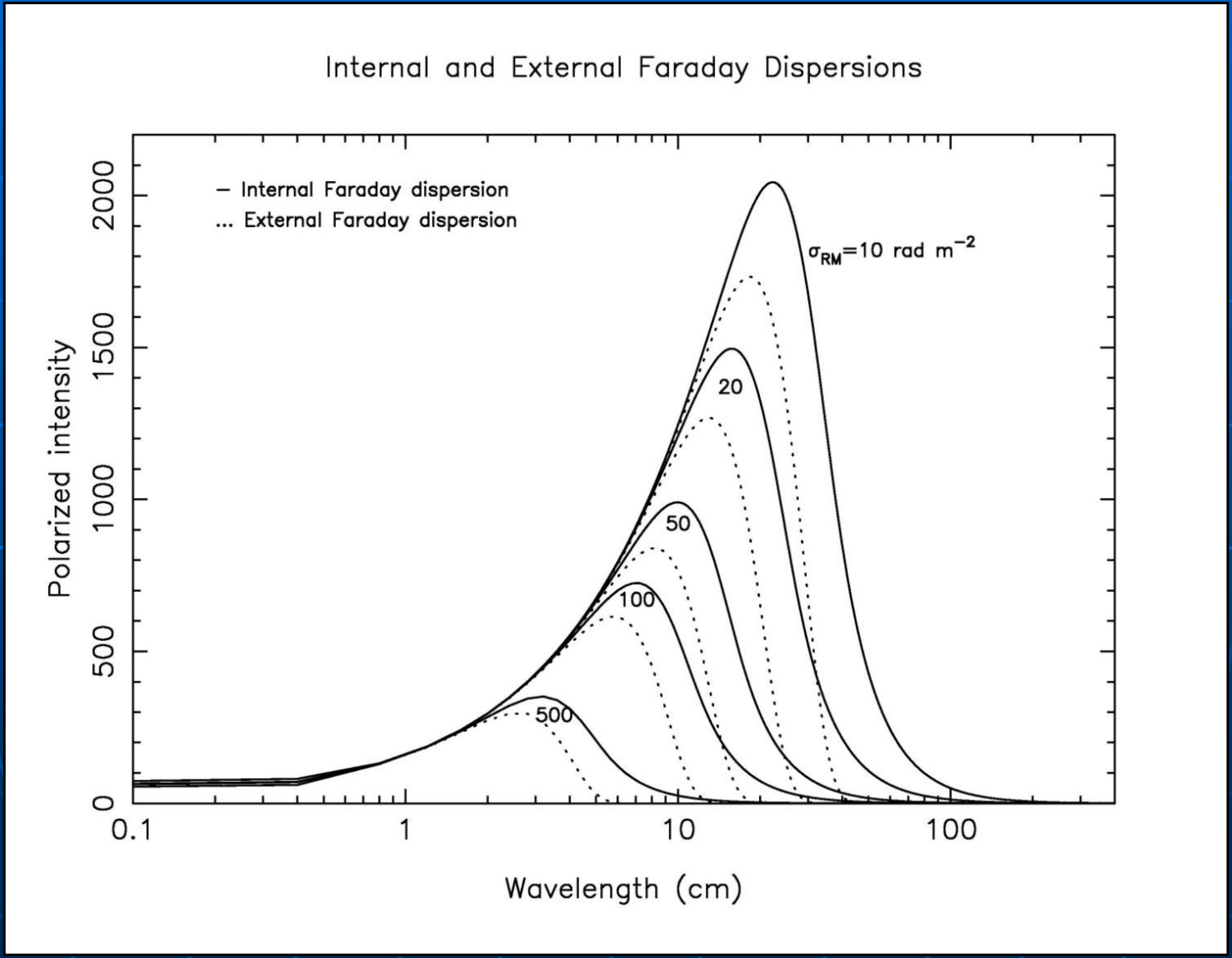


Internal Faraday dispersion
(wavelength-dependent)

Fletcher et al. 2004

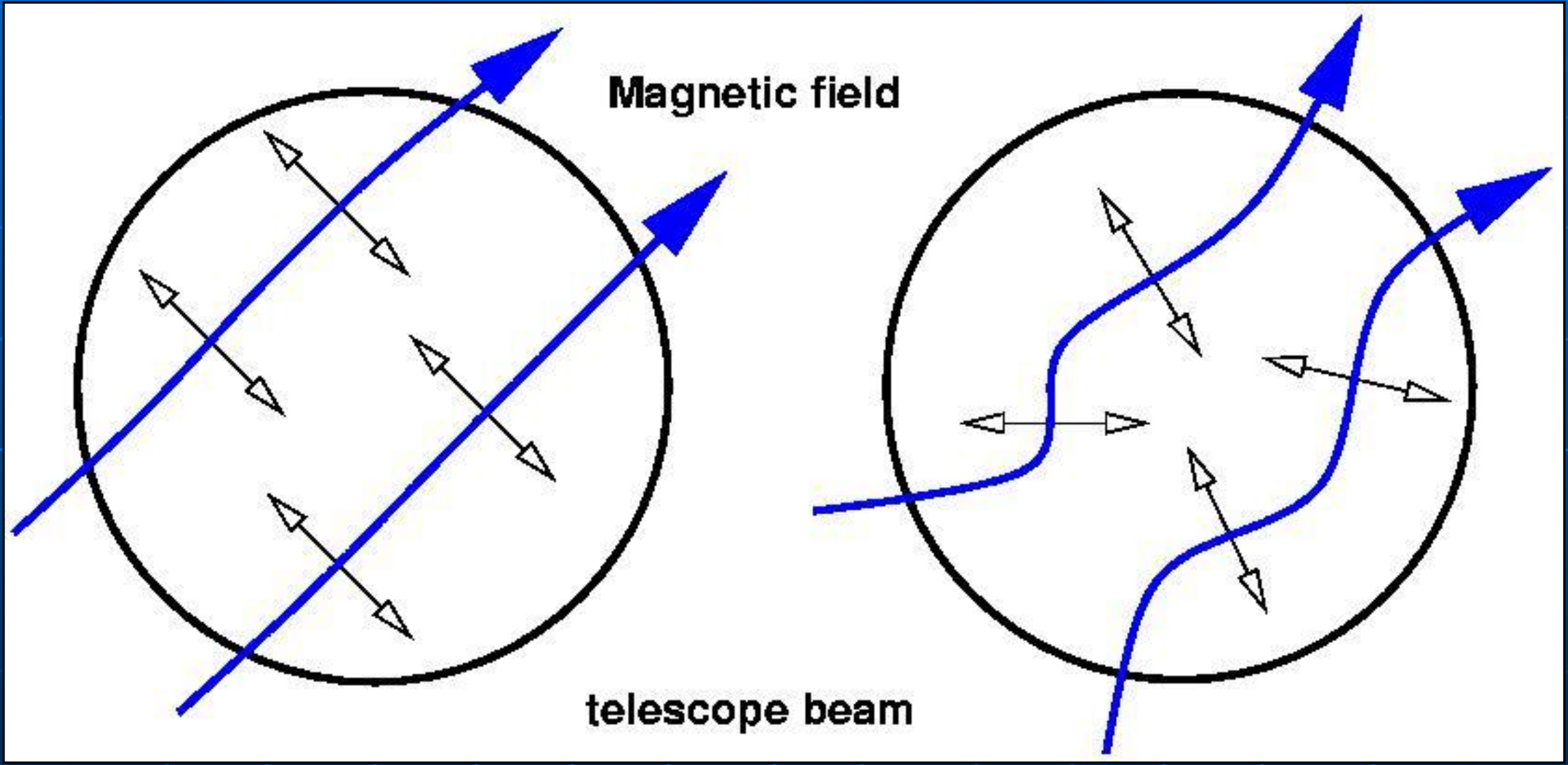


Maximum polarization



Beam depolarization
(wavelength-independent)

Fletcher et al. 2004



Depolarization effects

- Differential Faraday rotation (wavelength-dep.):

$$p = p_o \, |\sin(2\,RM\,\lambda^2)| \, / \, (2\,RM\,\lambda^2)$$

- Internal Faraday dispersion (wavelength-dep.):

$$p = p_o \, (1 - \exp(-2\,\sigma_{RM}^2\,\lambda^4) \, / \, (2\,\sigma_{RM}^2\,\lambda^4)$$

- External Faraday dispersion (wavelength-dep.):

$$p = p_o \exp(-2\,\sigma_{RM}^2\,\lambda^4)$$

- Faraday dispersion:

$$\sigma_{RM}^2 = (0.81\,n_e\,B_r)^2\,L\,d\,f_v$$

(B_r is the turbulent field strength, *d* the size of the turbulent cells)

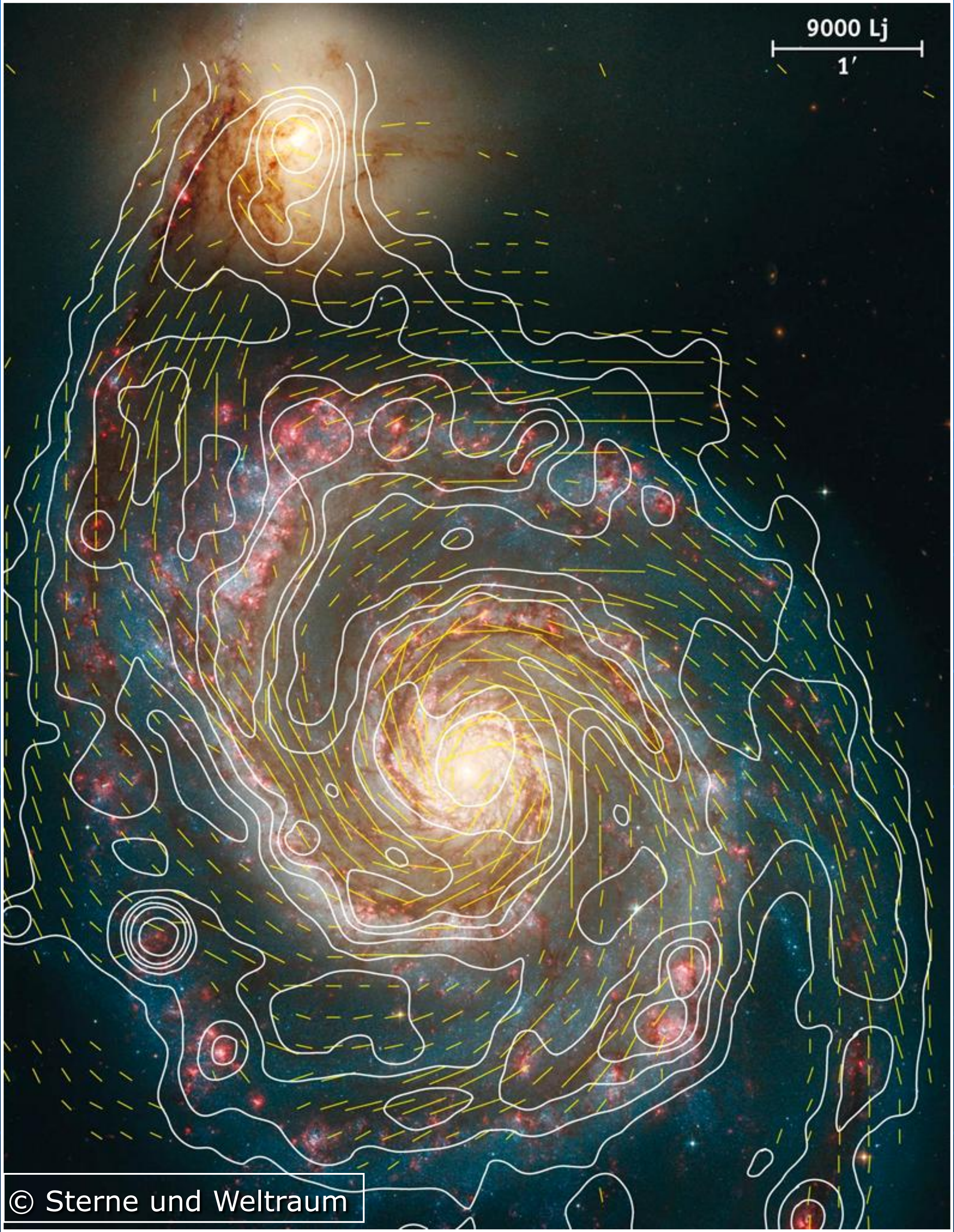
- Turbulent fields (wavelength-independent):

$$p = p_o \, / \, N^{1/2}$$

(where *N* is the number of turbulent field cells)

M 51

6cm VLA+Effelsberg
Total intensity
+ B-vectors
(Fletcher et al. 2010)

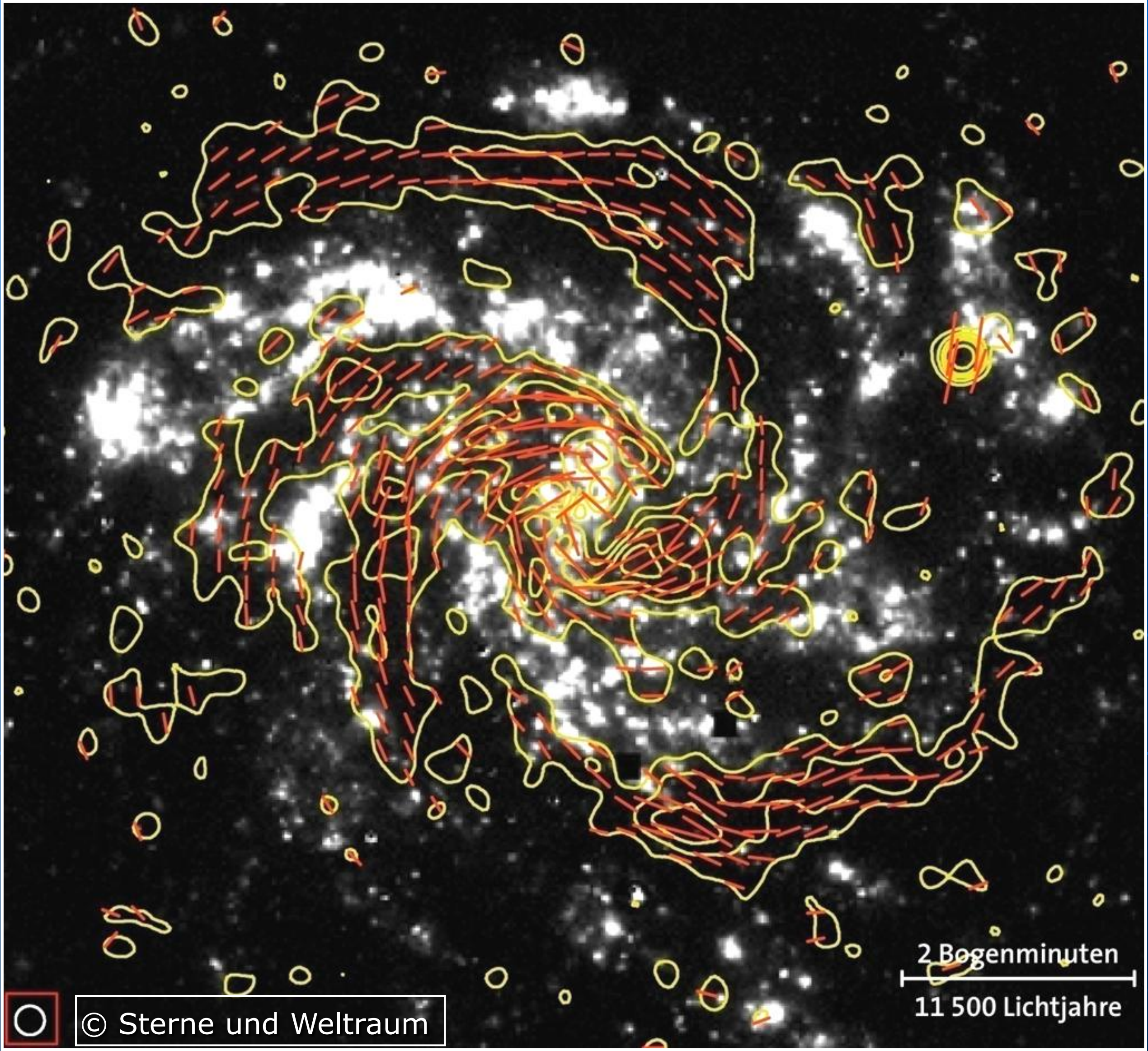


Small Faraday rotation:

Spiral fields are roughly parallel to the optical spiral arms

NGC 6946

6cm VLA+Effelsberg
Polarized intensity
+ B-vectors
(Beck & Hoernes 1996)

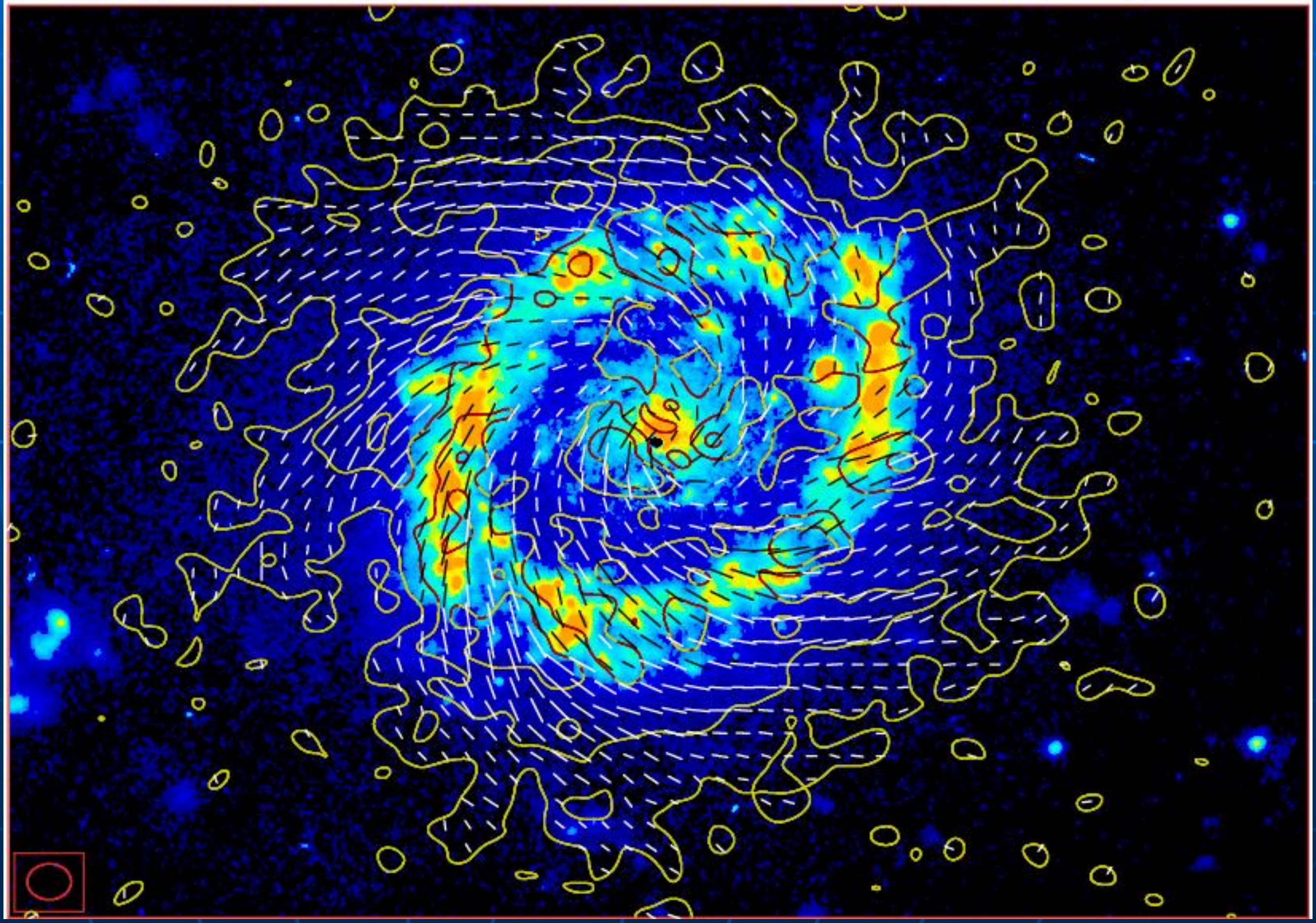


"Magnetic arms":

Ordered fields concentrated in interarm regions

NGC 4736

3cm VLA
Polarized intensity
+ B-vectors
(Chyzy & Buta 2007)

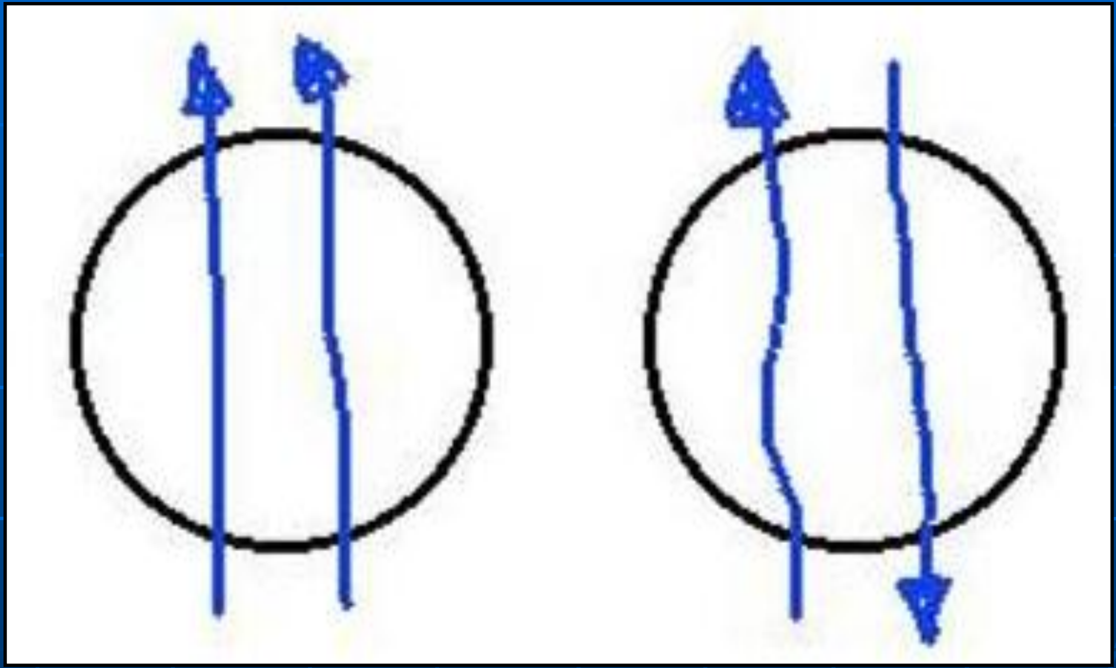


Spiral fields
in a
ring galaxy

Most spiral galaxies host spiral magnetic field patterns

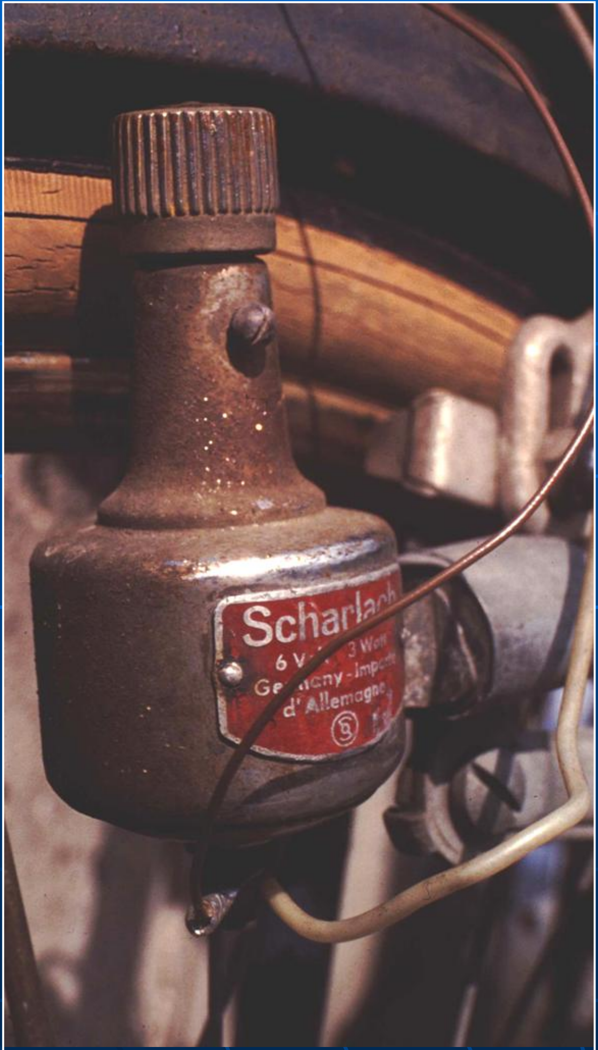
Fletcher et al. 2004

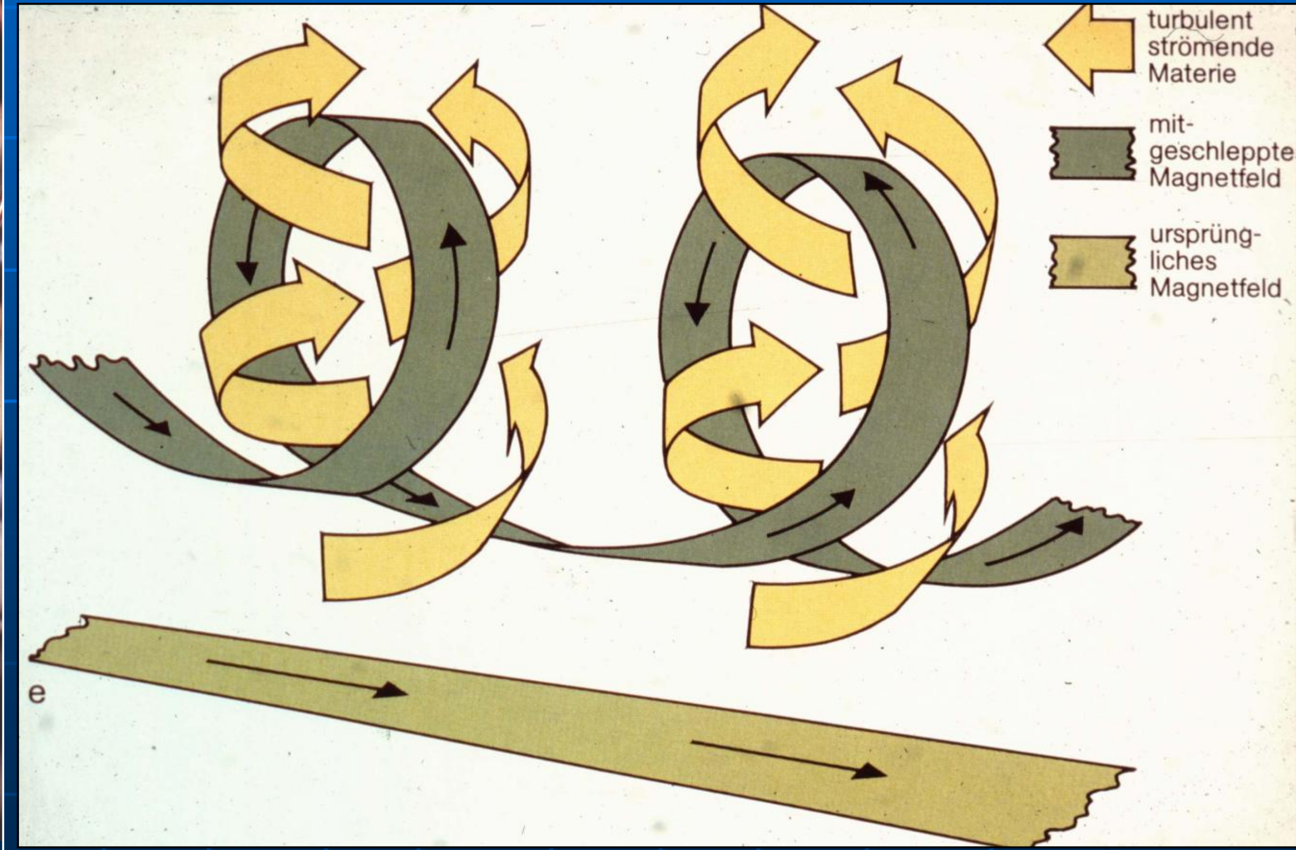
	Regular (coherent) field	Anisotropic (incoherent) field



Polarization :	strong	strong
Faraday rotation :	**high**	low

Dynamo theory

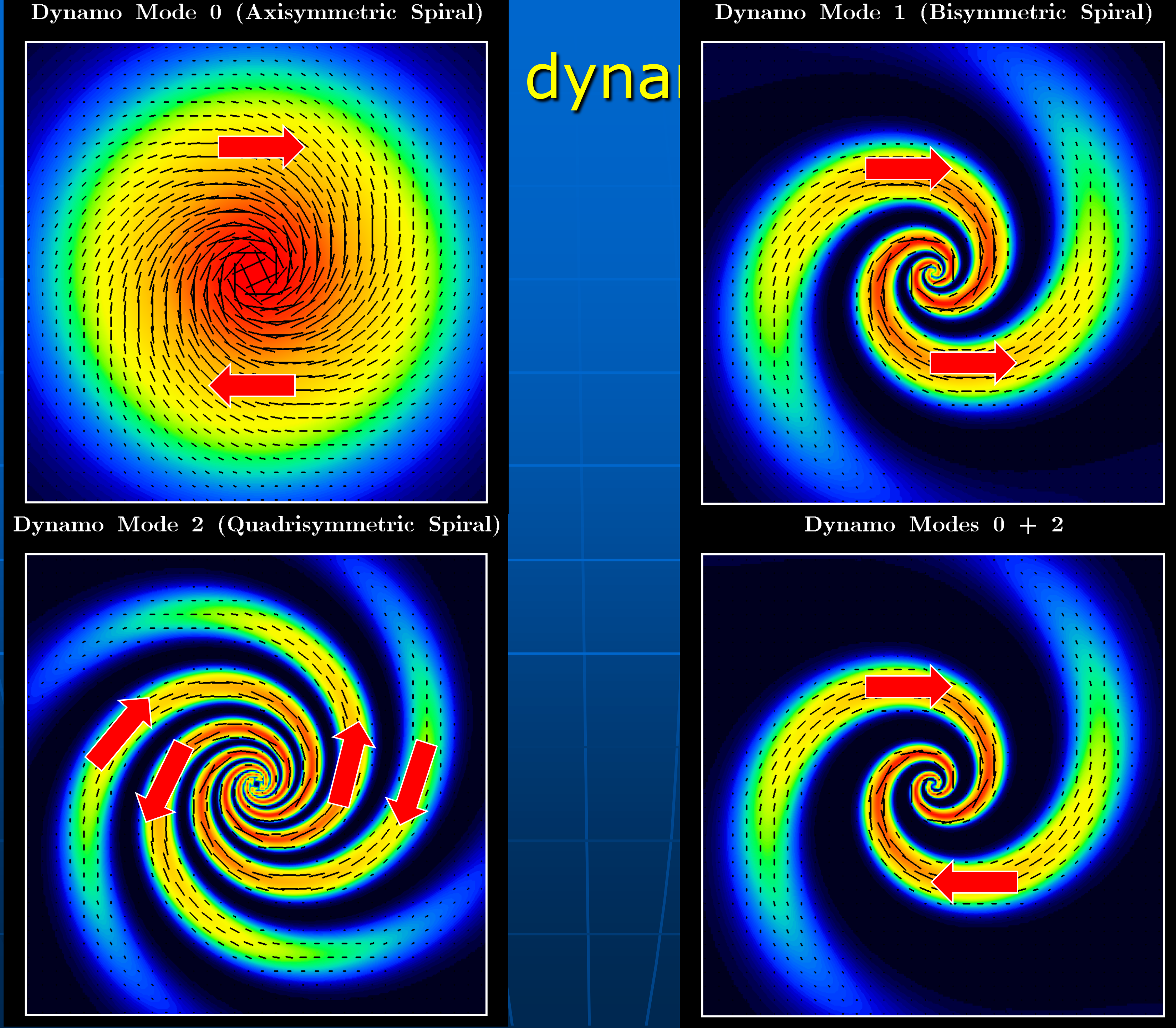




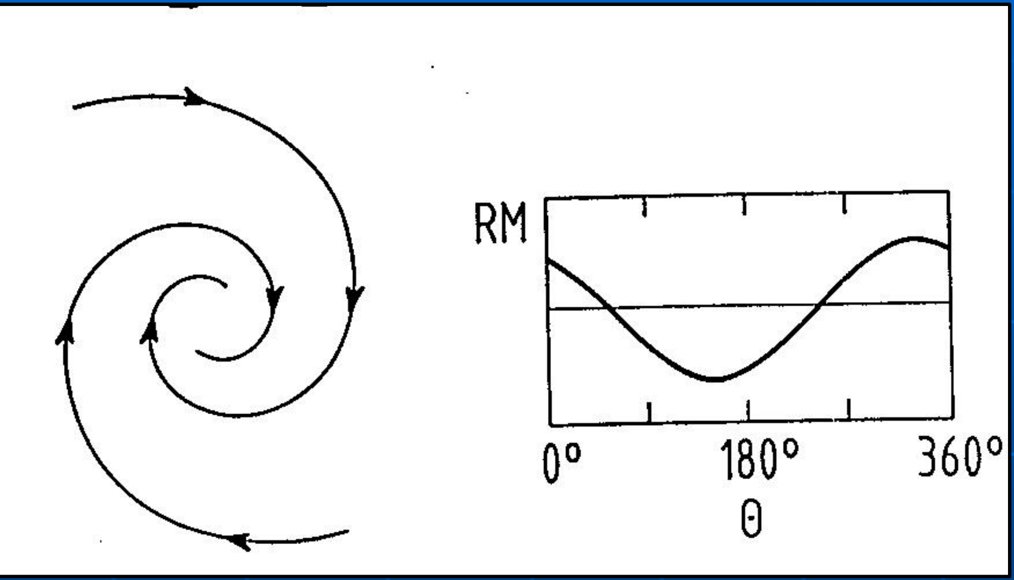
"Mean-field" dynamo theory for galactic fields

- Ingredients:
  Ionized gas + differential rotation + turbulence
- Dynamical separation between large scales and small scales
- Microphysics approximated by the average parameters
  "alpha-effect" (helicity) and magnetic diffusivity
- Fast reconnection needed to obtain the large-scale field
- Dynamo equation for the large-scale "mean" field
- Solutions: large-scale modes of regular (coherent) fields

dyna


Dynamo Mode 0 (Axisymmetric Spiral)
Dynamo Mode 1 (Bisymmetric Spiral)
Dynamo Mode 2 (Quadrisymmetric Spiral)
Dynamo Modes 0 + 2

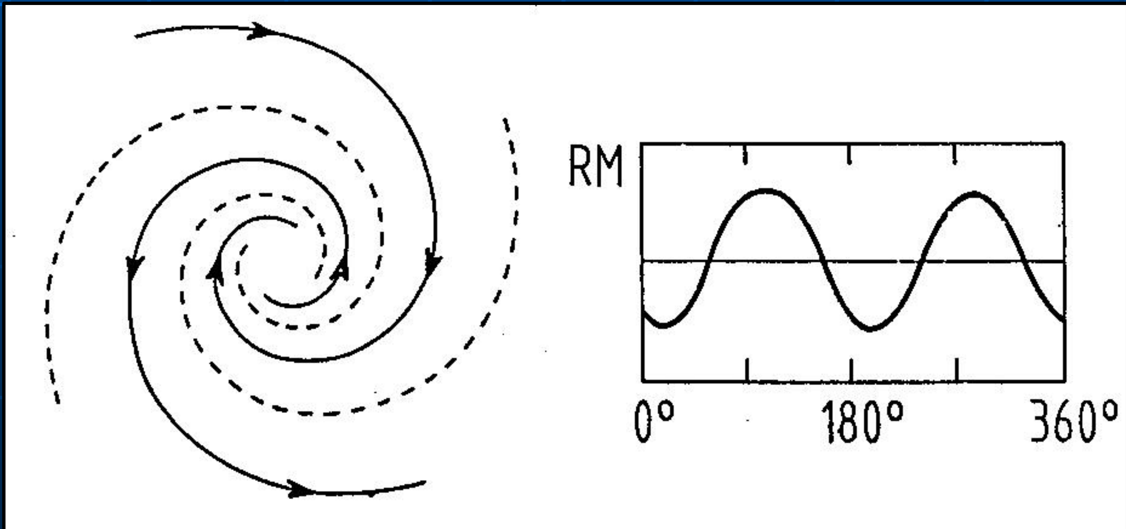
Finding dynamo modes:
Azimuthal variation of Faraday rotation



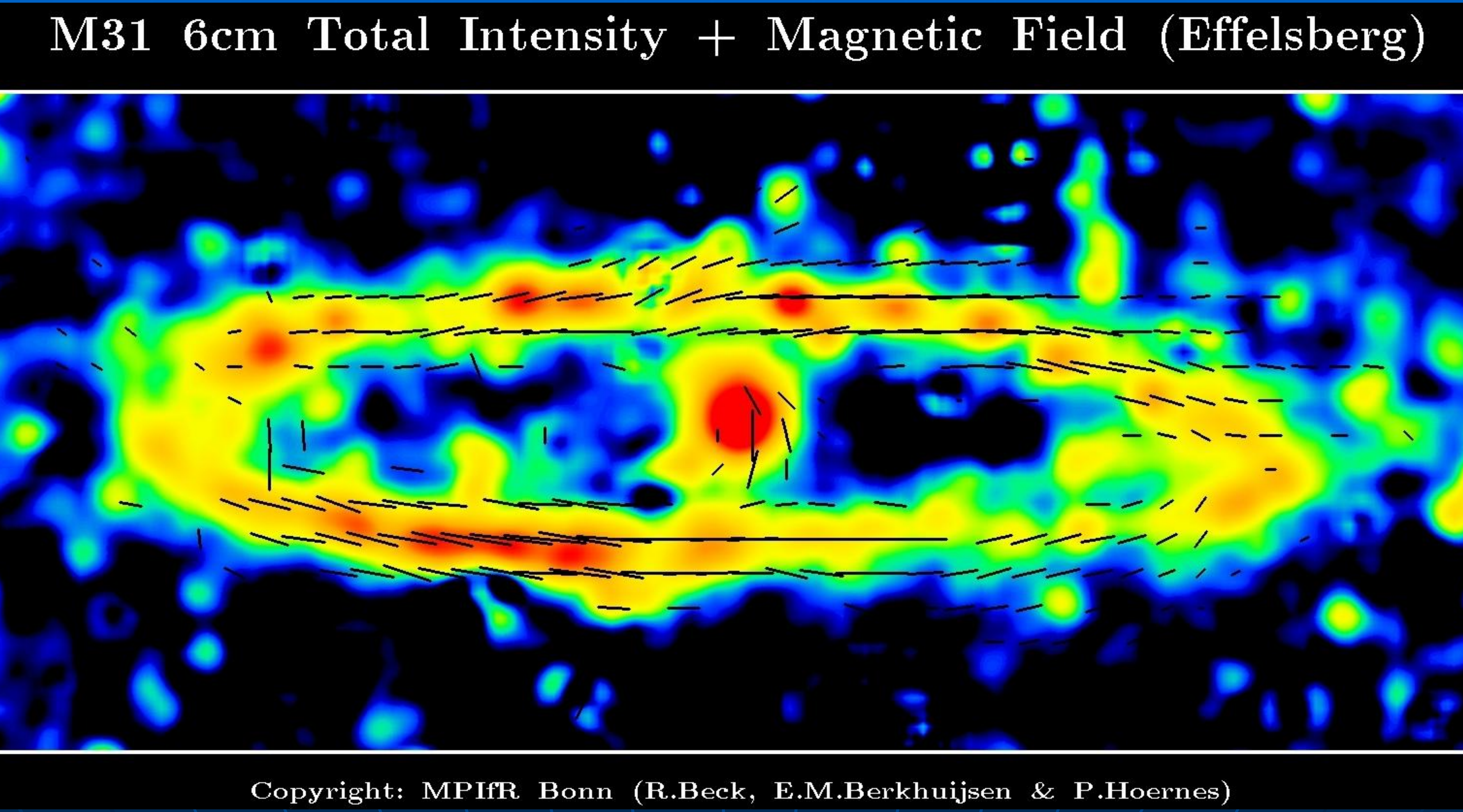
Krause 1990

Axisymmetric spiral
(m=0)

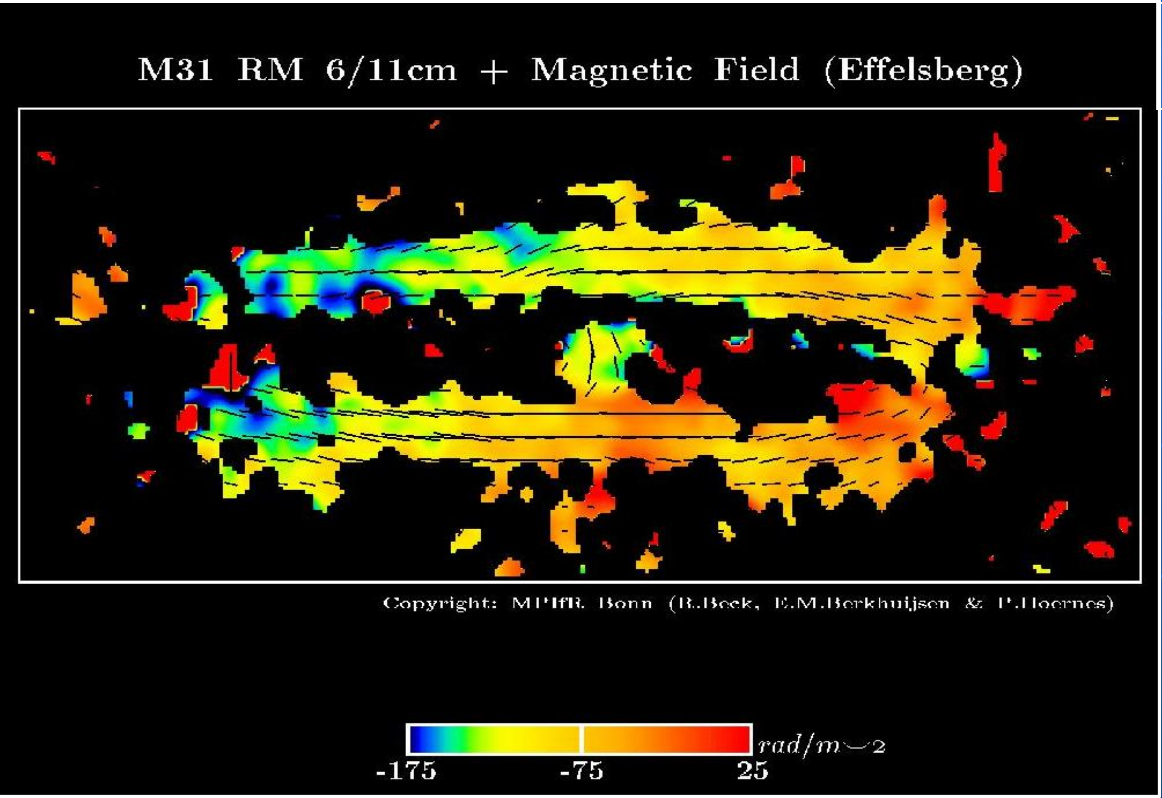
Bisymmetric spiral
(m=1)



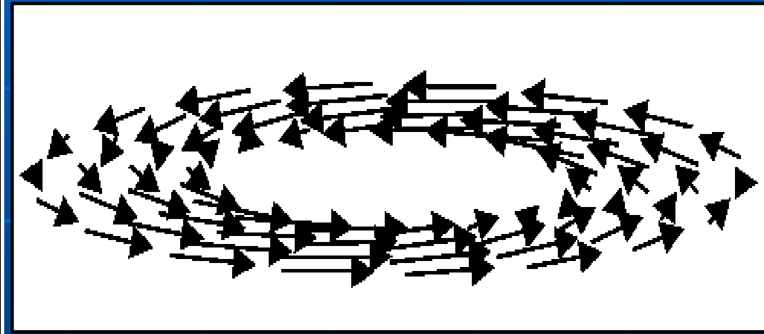
M31: The classical dynamo case



M31: The dynamo **IS** working !



Berkhuijsen et al. 2003



Fletcher et al. 2004

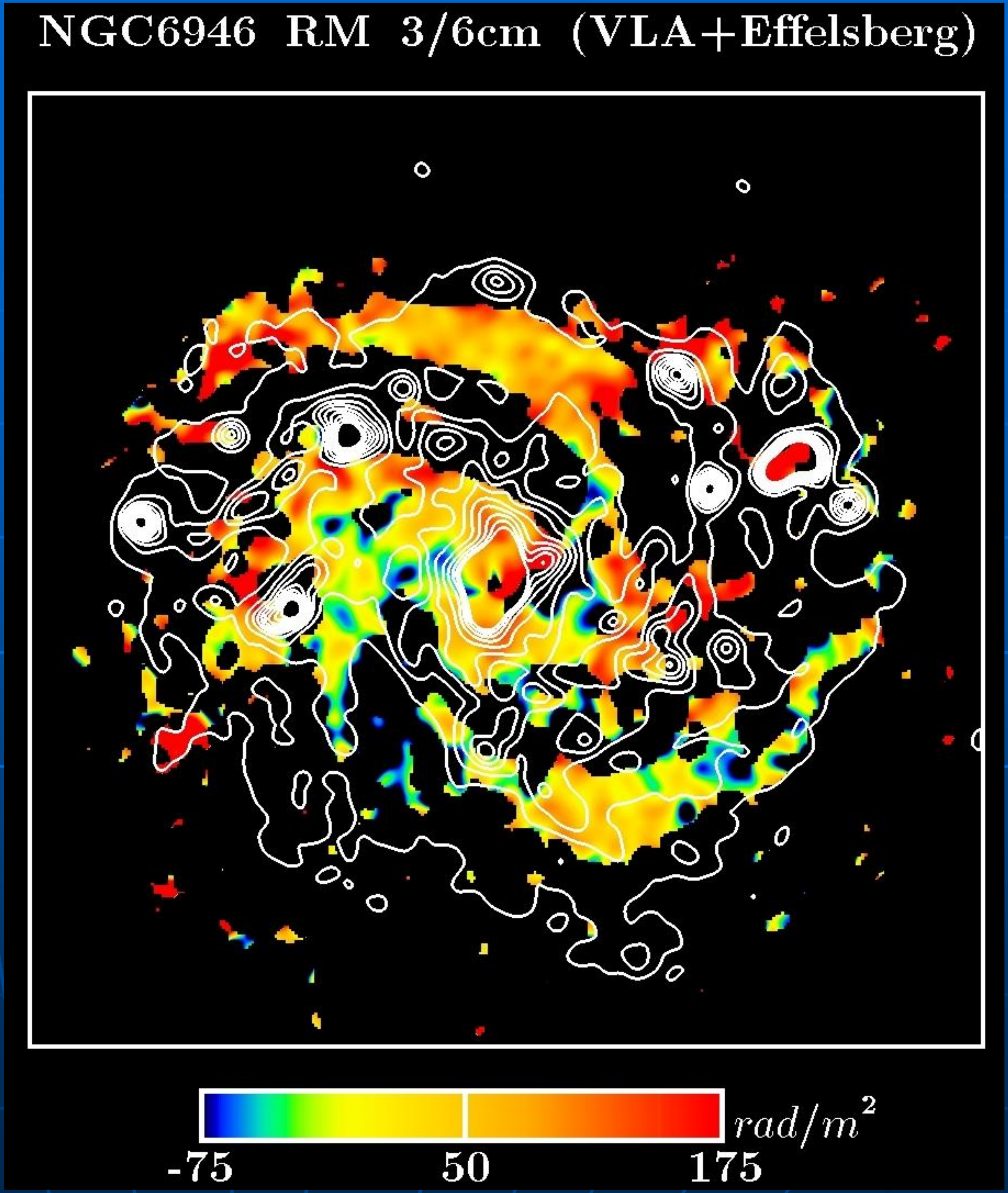
The spiral field of M31 is coherent and axisymmetric
(small spiral pitch angle, but no ring)

NGC 6946

RM 3/6cm
VLA+Effelsberg
(Beck 2007)

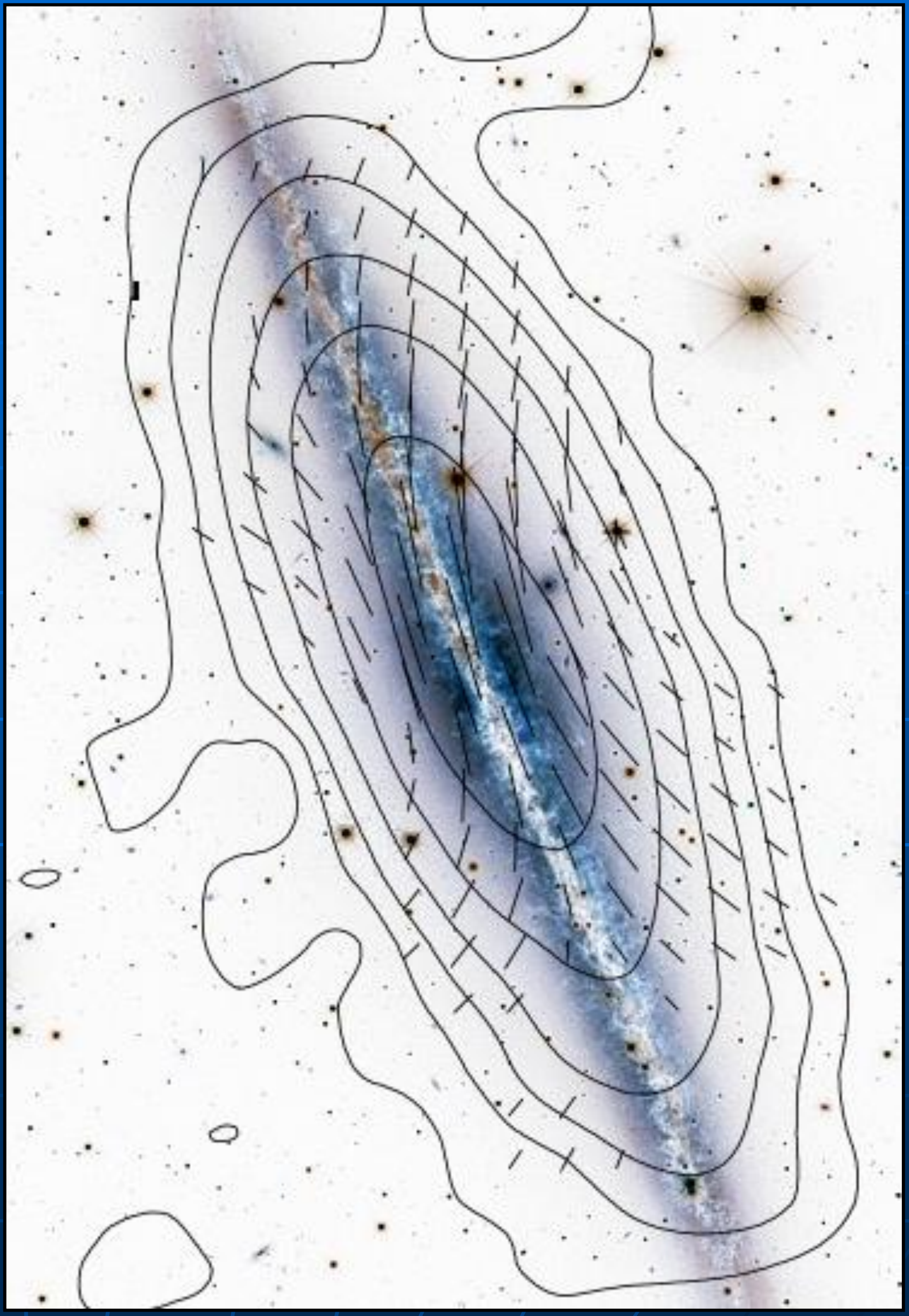
Inward-directed field:

Superposition of two dynamo modes (m=0 + m=2)



NGC 891

3cm Effelsberg
Total intensity
+ B-vectors
(Krause 2007)



Bright radio halo with X-shaped field pattern: quadrupolar dynamo field or driven by a disk wind?

Low-frequency radio observations

- Frequency of synchrotron emission: **$\upsilon \sim E^2 B$**

→ Observing at low frequencies traces cosmic-ray electrons in weak magnetic fields

- Lifetime of electrons due to synchrotron loss:

$t \sim \upsilon^{-0.5} B^{-1.5}$

→ Observing at low frequencies traces old electrons

- Faraday rotation: **$\Delta\psi \sim \upsilon^{-2}\, RM$**

→ Observing at low frequencies allows to measure small rotation measures

LOw Frequency ARray

LOFAR

10-80 MHz
110-240 MHz

Magnetism is key science

International Station Effelsberg

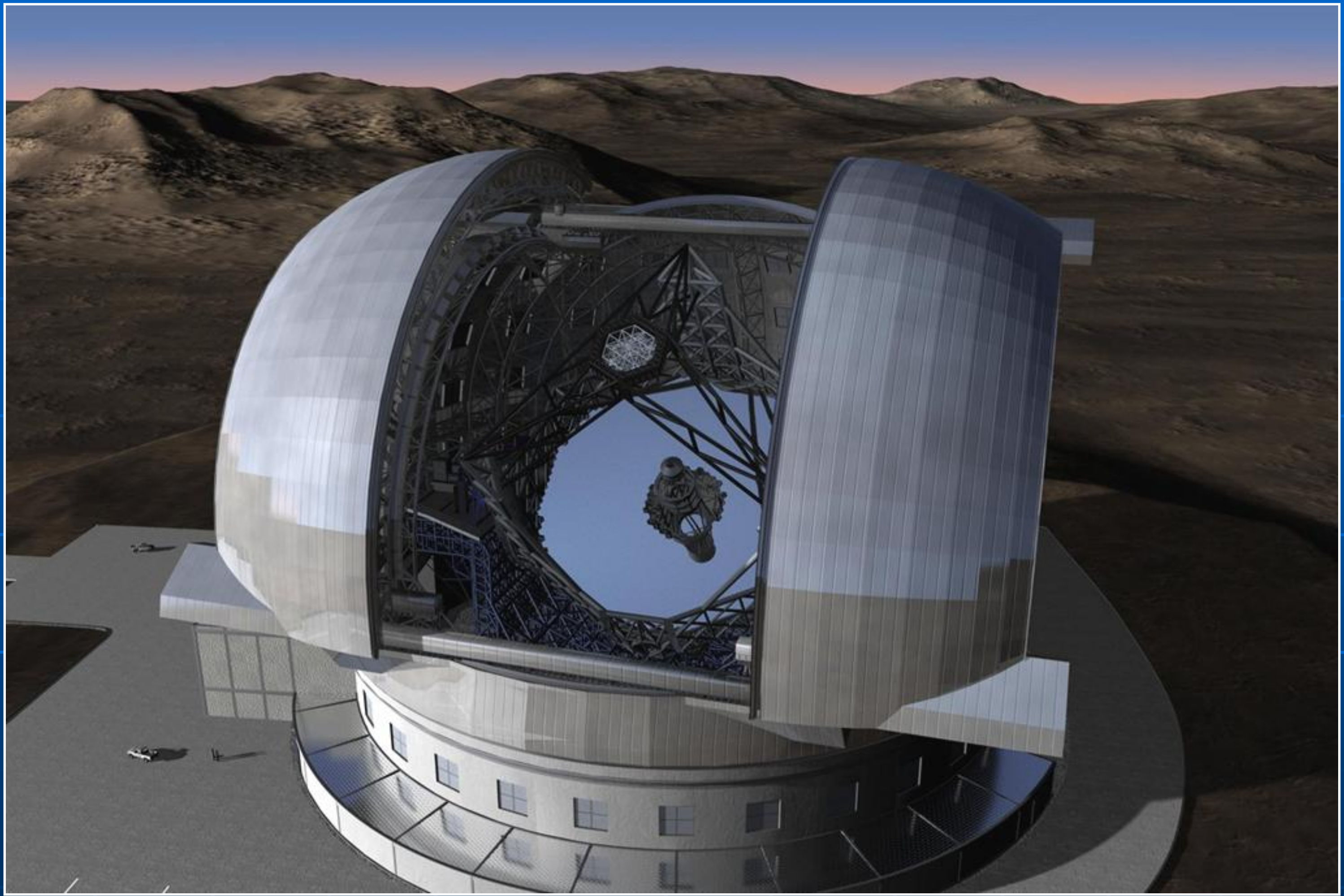
EVLA

- Frequency range: 1-50 GHz
- New correlator: 5-20x larger continuum sensitivity
- Spectro-polarimetry is now possible

ALMA

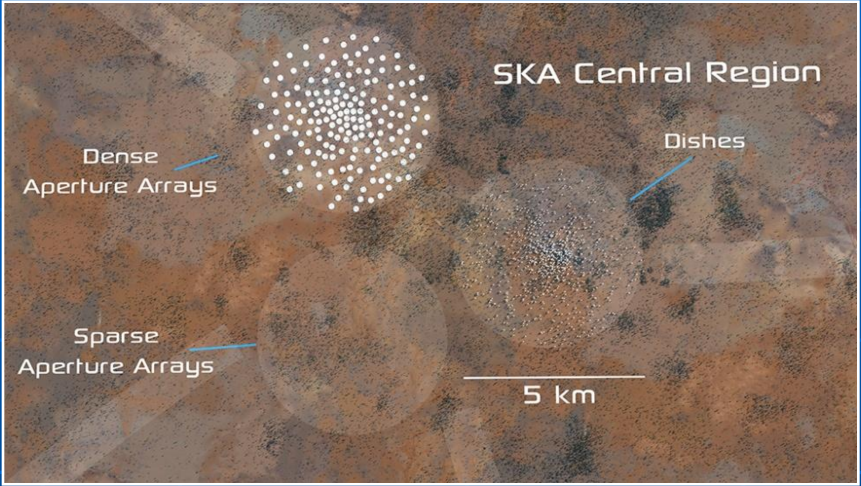
- Antennas: 54x 12-m plus 12x 7-m
- Receivers: 84-720 GHz (0.4-3.6mm), mostly with pol
- Construction: 2010-2015

E-ELT

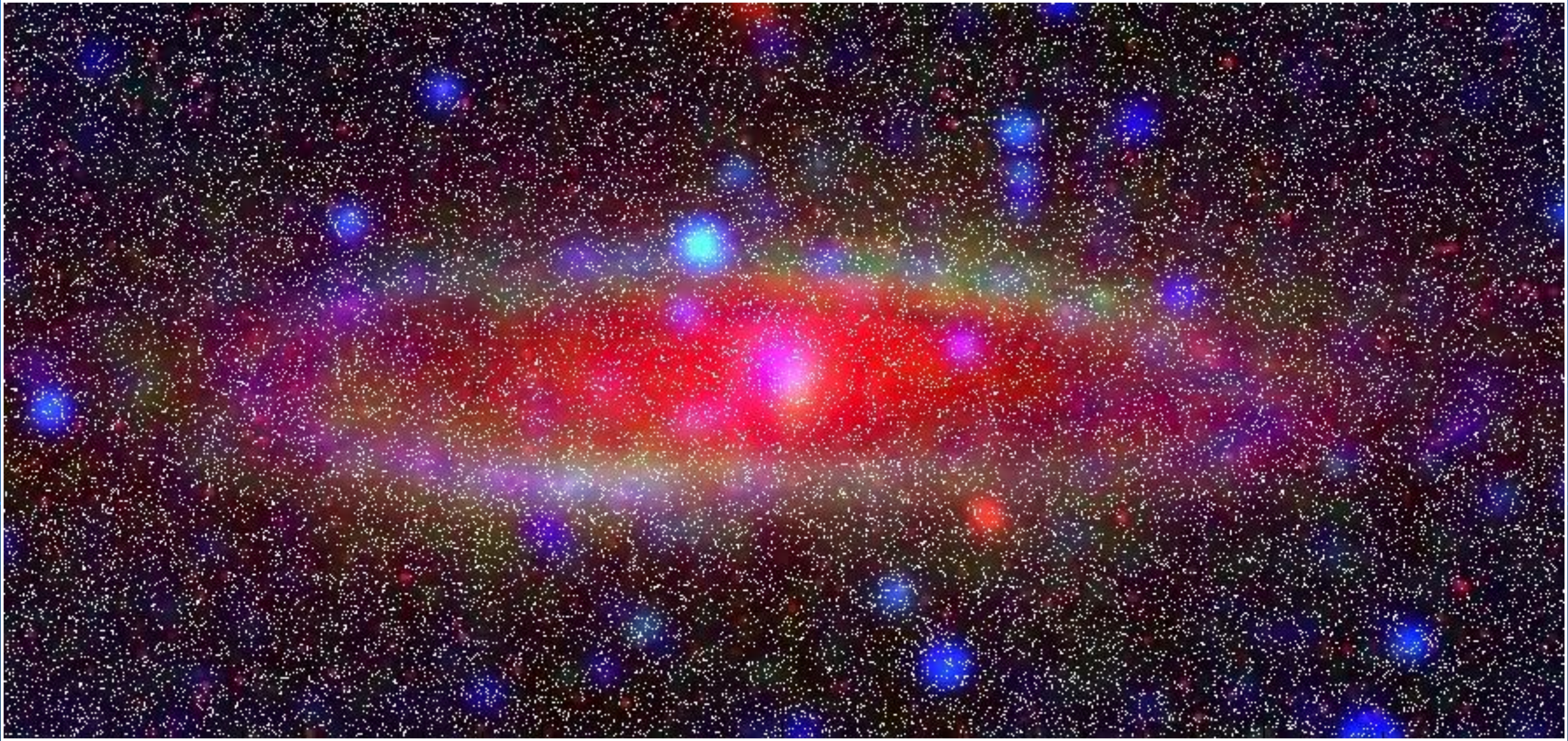


- Construction: 2012-2016
- Site: Cerro Armazones
- Polarimetric facilities still need efforts

SKA: full polarization
Magnetism is key science



SKA: RM grid of galaxies
(simulation by Bryan Gaensler)



≈10000 polarized sources shining through M31

Observation of magnetism in distant galaxies with the SKA

Murphy 2009