# Fundamentals in Radio Astronomy II

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# Content

- Black body and the Rayleigh-Jeans approximation
- Basics of radio telescopes and the radio radiation
- Exploring the cold universe through a warm window
- The radio astronomical receiver and the radiometer equation
- Sensitivity of a radio telescope
- Single dish vs. radio interferometers
- Selection of science by radio interferometer: an example

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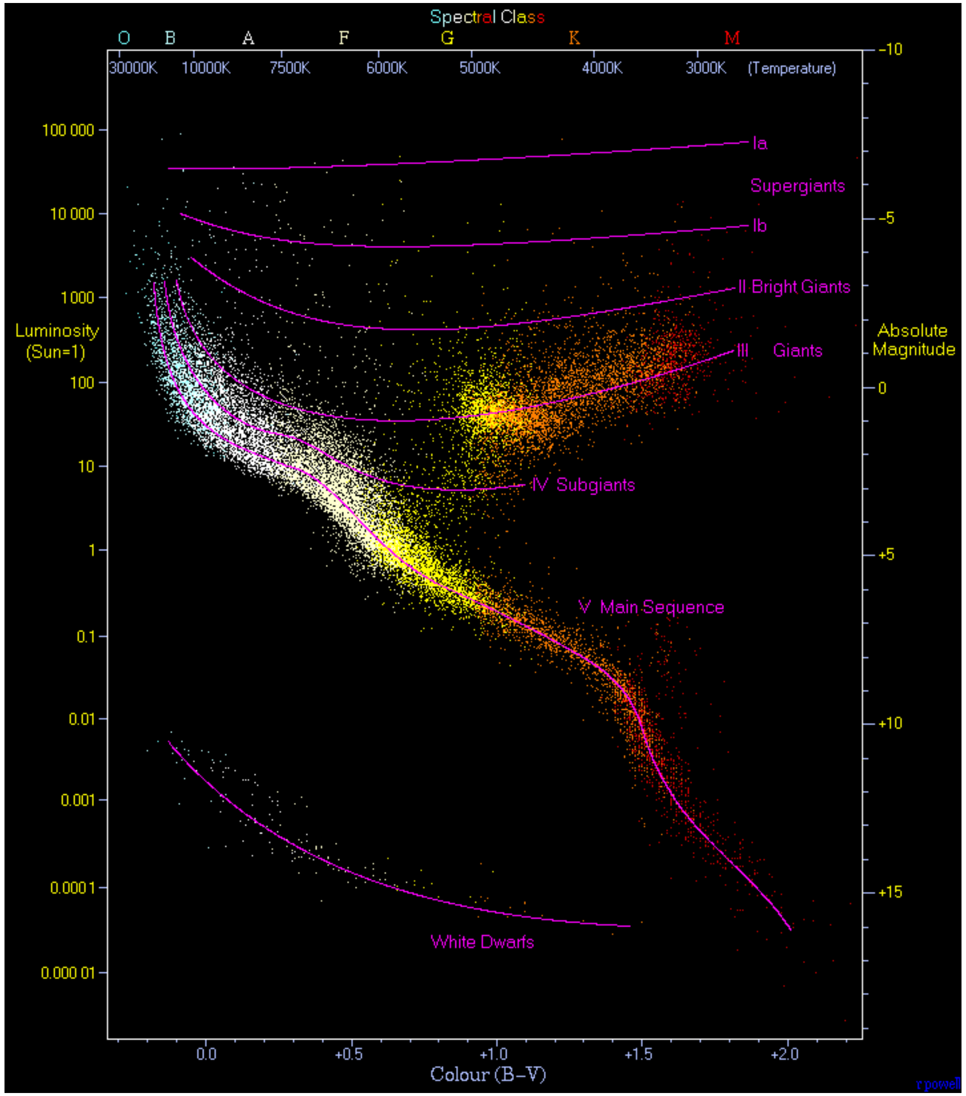
# Black Body and Rayleigh-Jeans Approximation

# Hertzsprung-Russel Diagram

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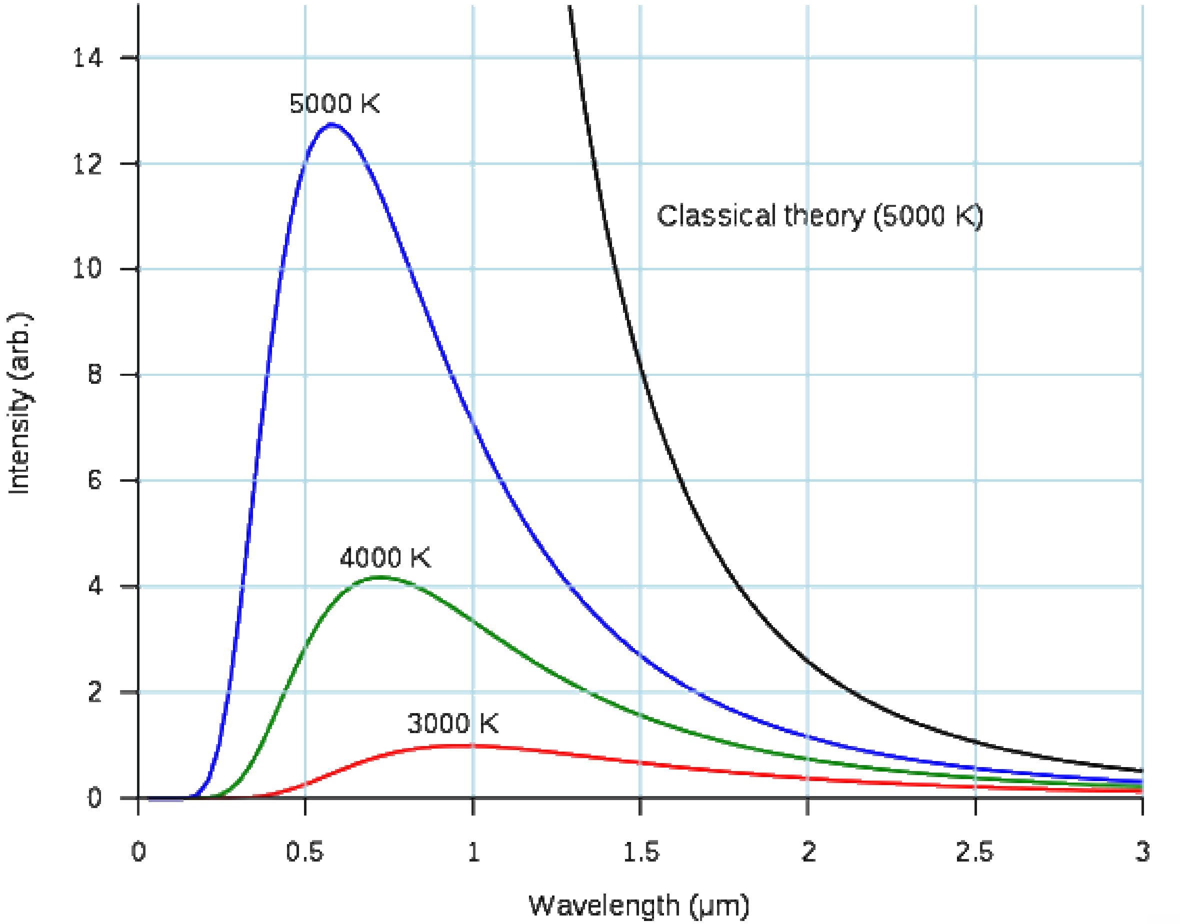
# Black Body Radiation

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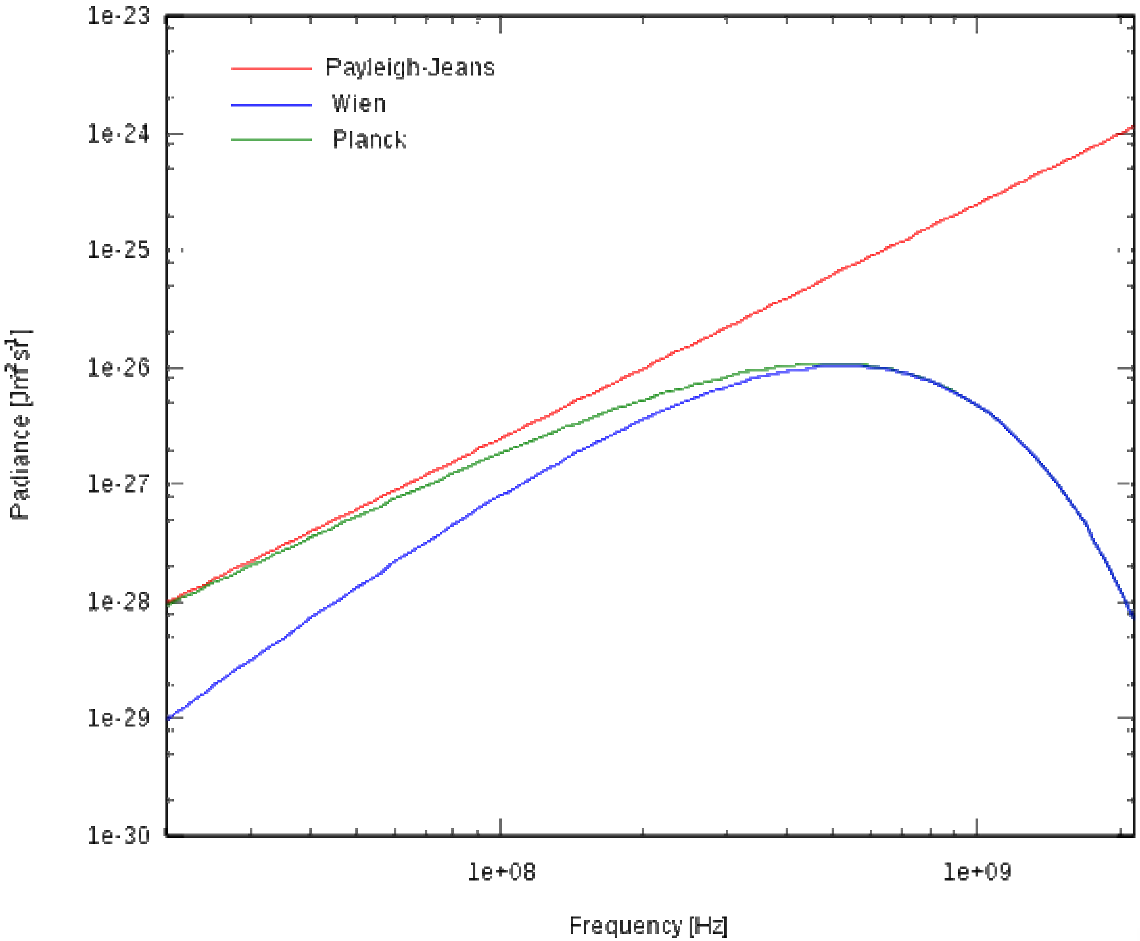
# Rayleigh-Jeans Approximation

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# Rayleigh-Jeans Approximation

- Consequences:
  - Radio astronomical **photons carry tiny amounts of energy**
  - Their **wavelengths are much longer than the size of atoms**, molecules and dust within the interstellar/intergalactic medium as well as the Earth atmosphere. Accordingly **Rayleigh-Scattering** ($\lambda \gg d$) is of marginal importance.
    - The radio sky is always dark

# The Radio Sky

http://www.cv.nrao.edu

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# Rayleigh-Jeans Approximation

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    - The radio sky is always dark
  - The low photon energy allows to apply **Gaussian statistics**! We always detect with radio astronomical receivers a high number of photons
  - The low photon energy makes the life easy, because we can approximate the Planck law via the **Rayleigh-Jeans** Ansatz

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# Rayleigh-Jeans Approximation

$$B_\nu(T) = \frac{2\mathrm{h}\nu^3}{\mathrm{c}^2} \cdot \frac{1}{e^{-\frac{\mathrm{h}\nu}{\mathrm{k}T}} - 1}$$

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$$e^{-\frac{h\nu}{kT}} - 1 \cong 1 + \frac{h\nu}{kT} + \ldots - 1 \cong \frac{h\nu}{kT} \Rightarrow$$

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$$B_\nu(T) = \frac{2\mathrm{h}\nu^3}{\mathrm{c}^2} \cdot \frac{\mathrm{k}T}{\mathrm{h}\nu} = \frac{2\mathrm{k}T\nu^2}{\mathrm{c}^2}$$

$$S_\nu = \int_{source} B(\theta, \Phi)\, d\Omega$$

# Basics of radio telescopes and the radio radiation

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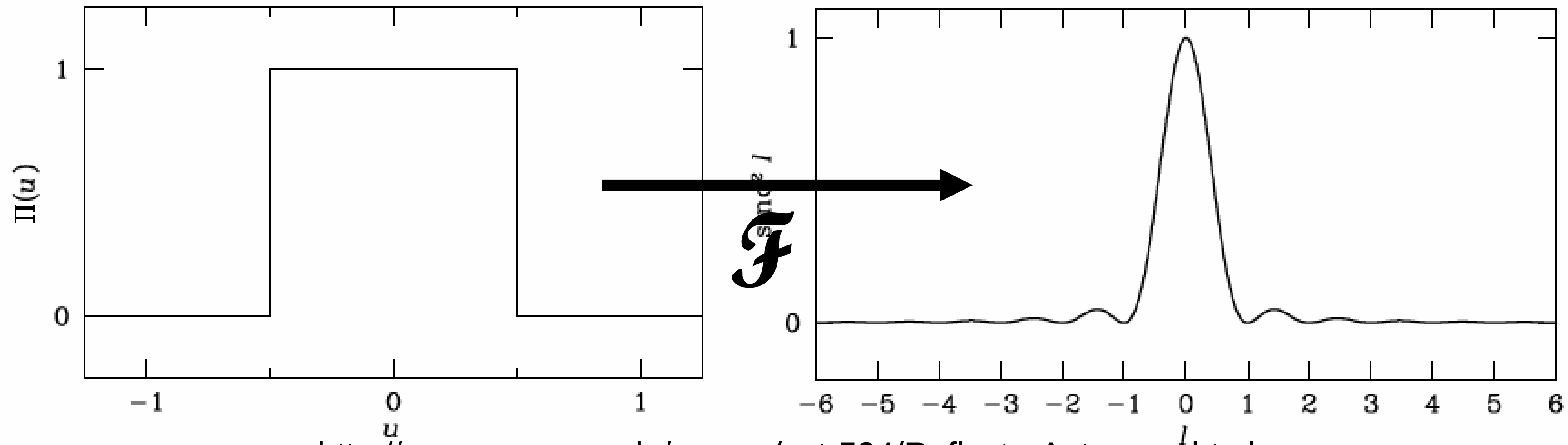
# Essentials I

- The separation between radio source and telescope is extremely large!
  - We can apply the „far field“ approximation, that makes the life pretty easy

# Far Field Approximation




http://www.cv.nrao.edu/course/astr534/ReflectorAntennas.html

- **Left**: An ideal telescope with a constant sensitivity across the whole aperture. No secondary mirror degrades the illumination of the primary reflector.
- **Right**: The image of an unresolved object yield for an ideal telescope aperture the so-called Airy-pattern. The with of the central maximum and the separation between the 2nd order maxima depend of the wavelength and define the angular resolution of a telescope ($\theta \approx \lambda/D$)

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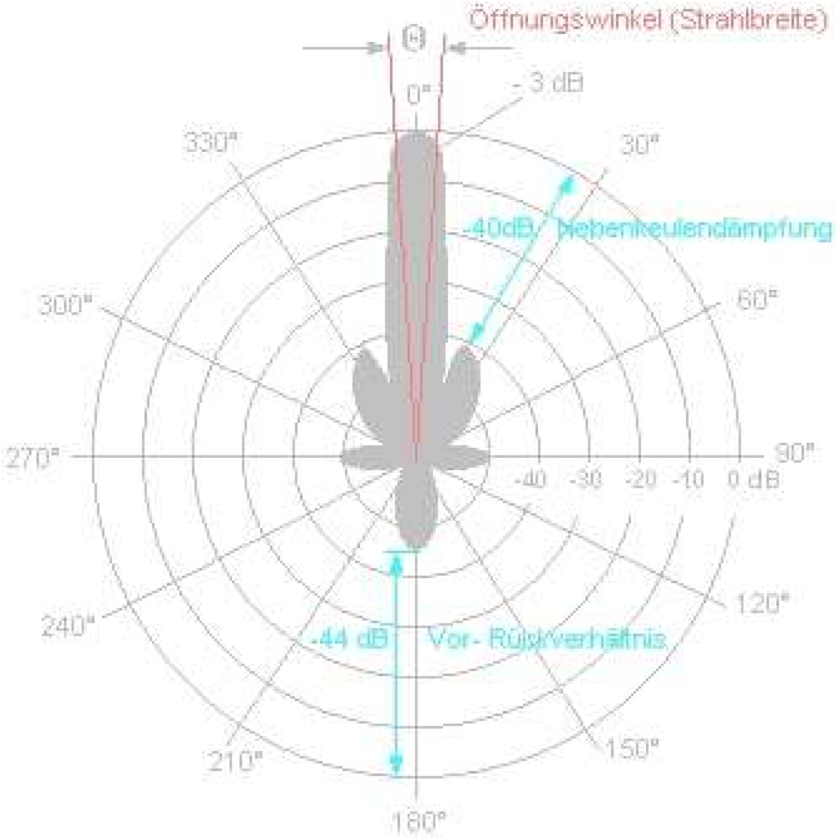
# Far Field Approximations

- The sensitivity pattern of a real radio telescope is a 3-D structure. The main beam defines the angular range with the highest sensitivity. The width of the main beam depends on the size of the telescope and the observational wavelength.
- The relative sensitivity between the main beam and the so-called side lobes (Airy pattern) is determined by the geometry of the radio dish. Typically about 72% of the total power received by the telescope enters the system via the main beam




http://www.stsm.info/dipl/dipl_3.html

$$\theta_{HPBW} \cong 1.22 * \frac{\lambda}{D}$$

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# Essentials II

- The separation between radio source and telescope is extremely large!
  - We can apply the „far field“ approximation, that makes the life pretty easy
- In radio astronomy we always sample coherent parts of the electromagnetic wave
  - We restore amplitude and phase of the wave

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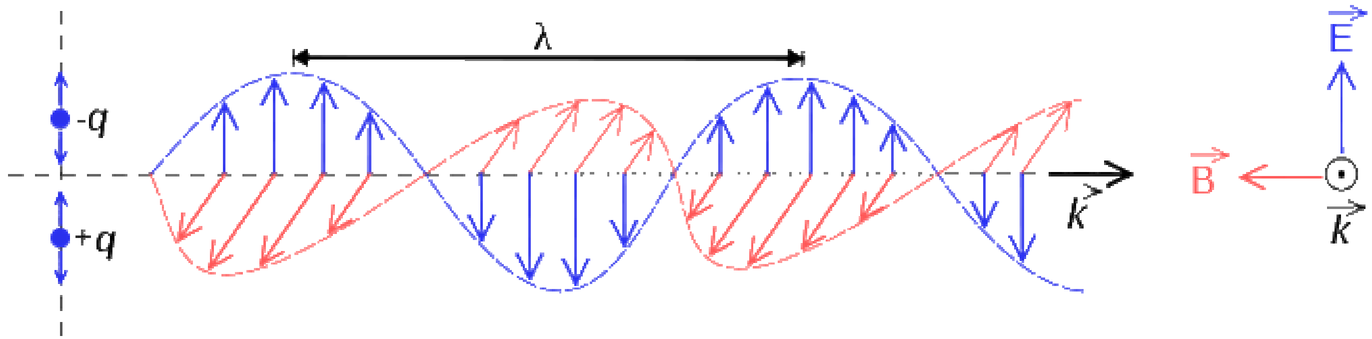
# Electromagnetic wave

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Information on the astronomical source we gain nearly exclusively via photons <-> electromagnetic waves

The Poynting-vector describes the density and direction of the electromagnetic wave.

In radio astronomy we measure the amplitude and the phase of each individual wave front.

In radio astronomy we observe very often „forbidden transitions“ leading to long life times $\tau$ and hence $c \cdot \tau = l_c$ long coherence times.

# Essentials III

- The separation between radio source and telescope is extremely large!
  - We can apply the „far field“ approximation, that makes the life pretty easy
- In radio astronomy we always sample coherent parts of the electromagnetic wave
  - We restore amplitude and phase of the wave
- Radio waves trace the cold phase of the Universe

$$\mathrm{k}T = \mathrm{h}\nu \Rightarrow T = \frac{\mathrm{h}\nu}{\mathrm{k}} = \frac{6.67\cdot 10^{-34}\,\mathrm{J}\cdot\mathrm{s} * 1\cdot 10^{9}\,\mathrm{Hz}}{1.38\cdot 10^{-23}\,\mathrm{J}\cdot\mathrm{K}^{-1}} \approx 0.05\mathrm{K}$$

$$T(450\mathrm{nm}) = \frac{6.67\cdot 10^{-34}\,\mathrm{J}\cdot\mathrm{s} * 6.7\cdot 10^{14}\,\mathrm{Hz}}{1.38\cdot 10^{-23}\,\mathrm{J}\cdot\mathrm{K}^{-1}} \approx 32.000\mathrm{K}$$

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# Exploring the cold universe through a warm window

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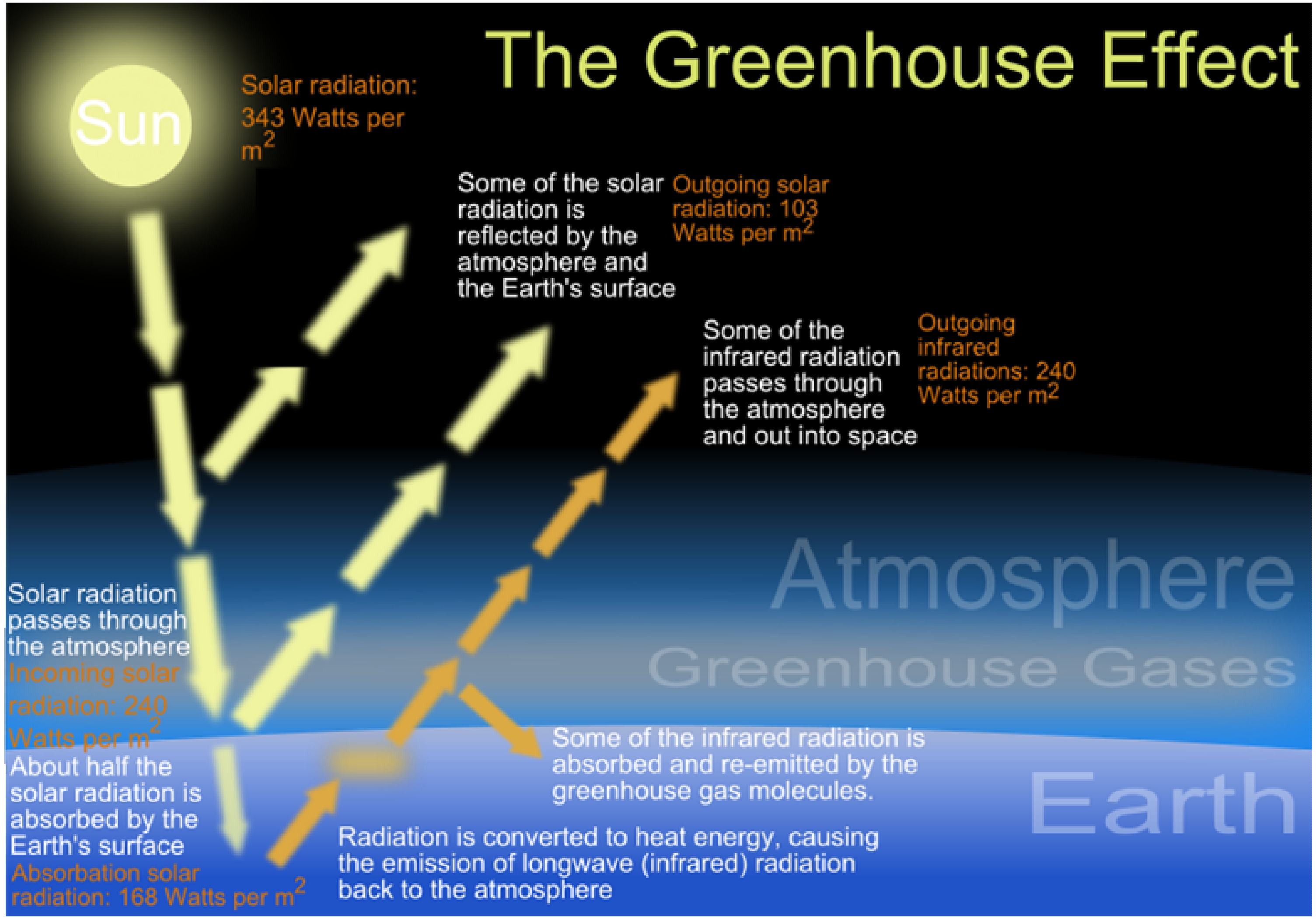
# The Earth atmosphere

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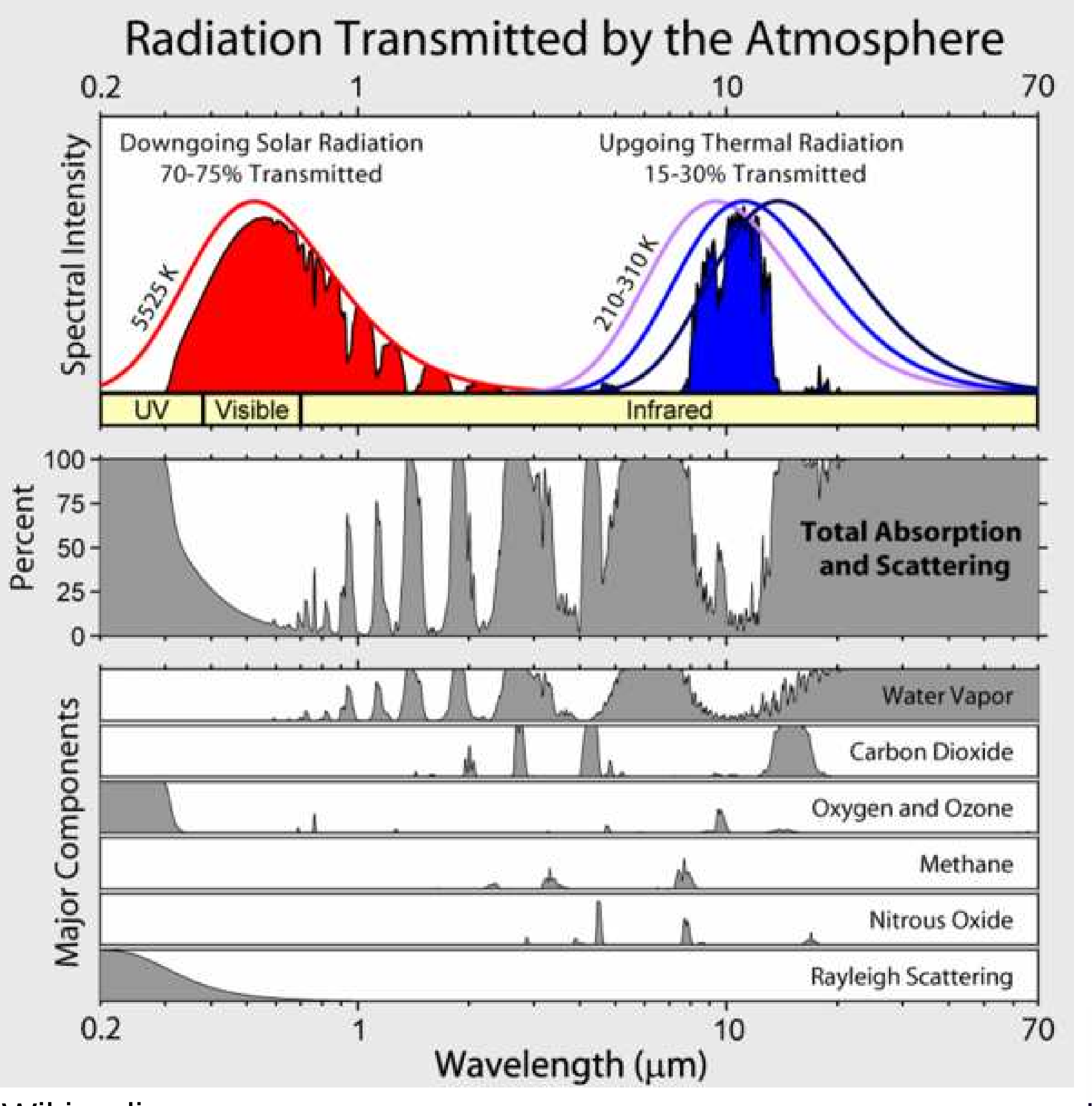
# Energy gain and loss

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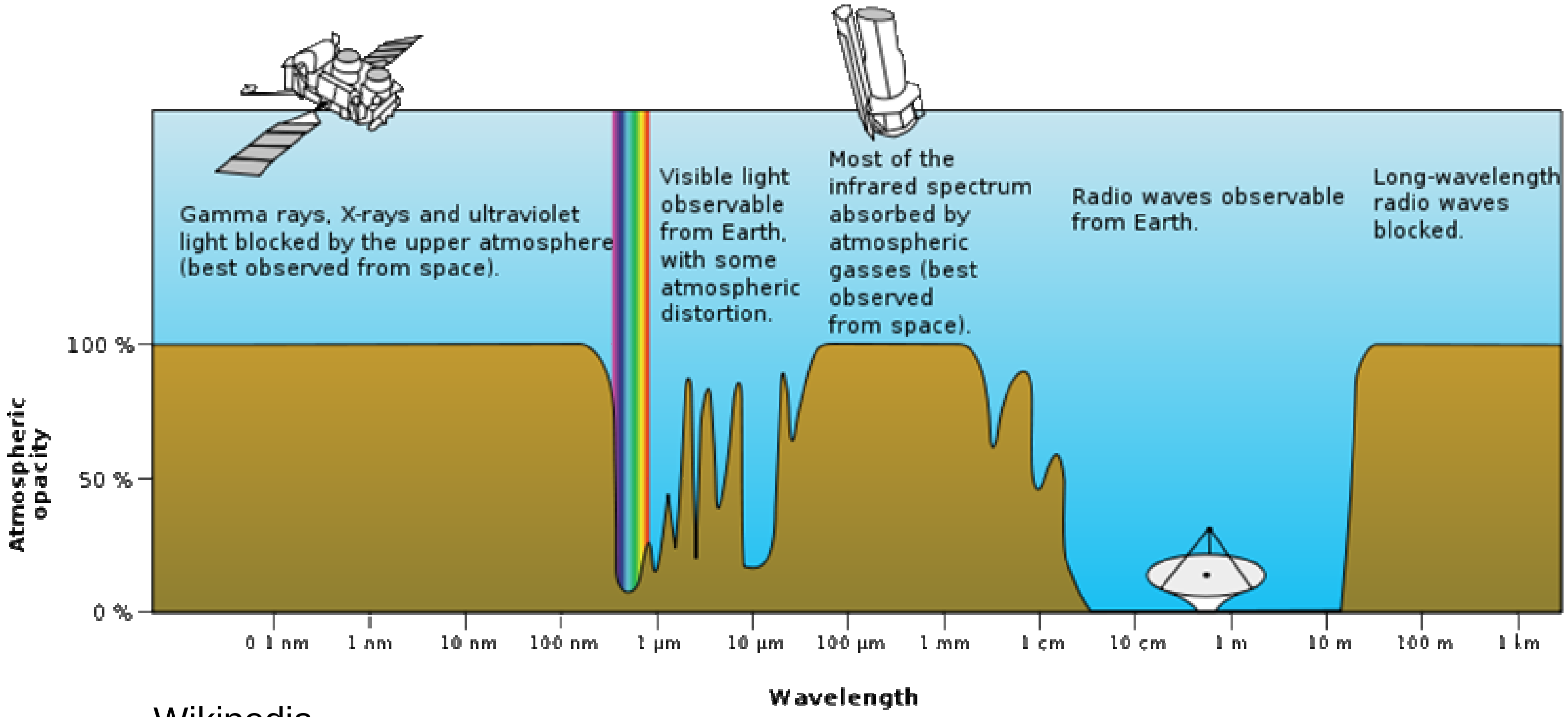
# Windows to the Universe

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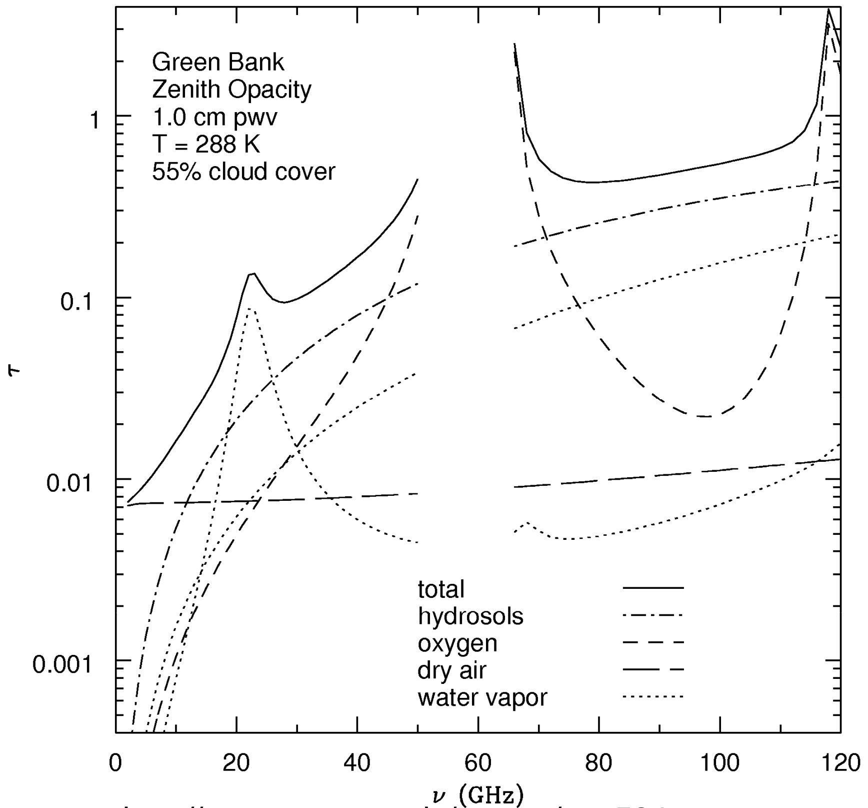
# Attenuation of radiation

- The composition, temperature and climatic condition of the radio **astronomical site** is of high importance for the scientific yield of a telescope
- Shown on the right hand side is the **opacity** of the earth atmosphere for a cm-wave radio telescope at an low altitude site
- **Molecular oxygen** has a permanent magnetically dipole moment **between 52 and 68 GHz** and attenuates all radio radiation from space. The width of the atmospheric lines depend on the air pressure at the site.
- At **22 GHz** the attenuation is due to **water vapor**. The width of the line is about 4 GHz.
- **Hydrosols** are tiny raindrops (r ~ 0.1 mm) which scatter the radio wave following the Rayleigh scattering process.




http://www.cv.nrao.edu/course/astr534

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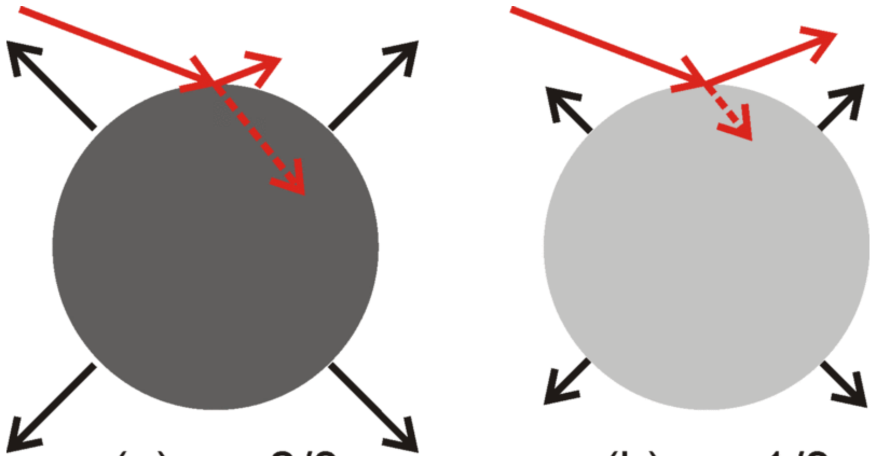
# Kirchhoff Law



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- According to the Kirchhoff law on thermal radiation exists a direct **relation between** the ability to **absorb** electromagnetic radiation and the **emission** of radiation.
- In **thermodynamic equilibrium** the absorbed amount of energy is **equal** to the re-radiated one!
- The molecules of the Earth **atmosphere** re-radiate the power of energy they gained from the **Sun**.
- This thermal radiation can be approximated by a black body of low temperature. This **radiation enters the receiver** of the radio dish and adds the thermal noise to the observed radiation of the astronomical source.
- **Seasons** are important for the quality of the radio astronomical measurement!

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# Consequences

$$T_{noise} = \frac{P_\nu}{\mathrm{k}}$$

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$$T_{sys} = T_{CMB} + T_{source} + T_{atmosphere} + T_{receiver} + \ldots$$

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$$T_{noise} = \frac{P_\nu}{\mathrm{k}}$$

$$T_{sys} = T_{CMB} + T_{source} + T_{atmosphere} + T_{receiver} + \ldots$$

$$T_{source} << T_{sys}$$

- Astronomical sources down to the mK/µK-level are detectable while the system temperature of the superposition of all thermal radiating sources is up to a few hundred Kelvin!

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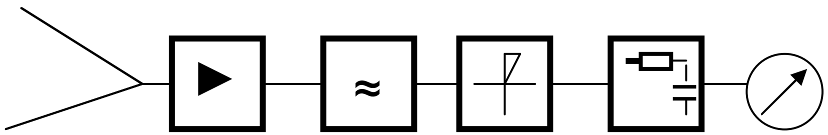
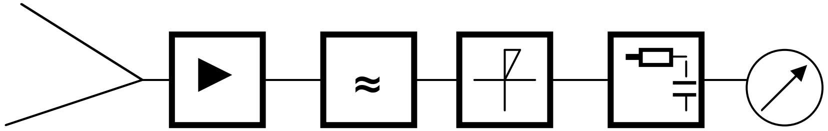
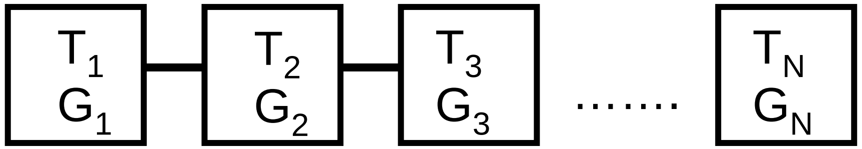
# The radioastronomical receiver and the radiometer equation

# $T_{receiver}$

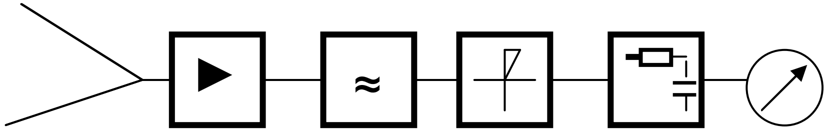


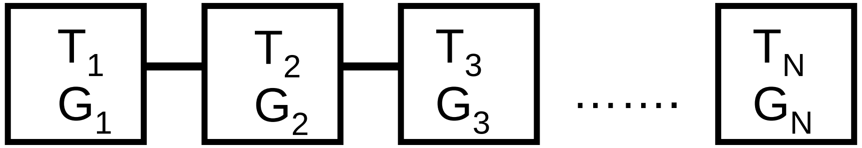
## $T_{receiver}$





## $T_{receiver}$

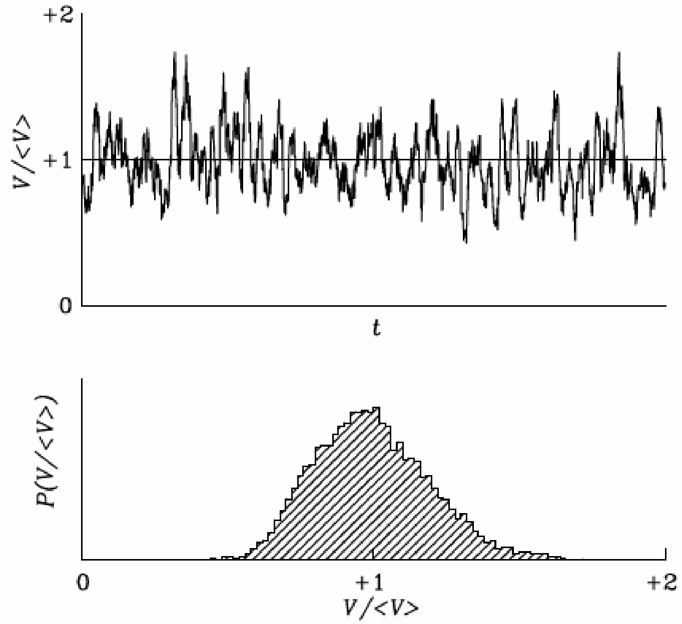


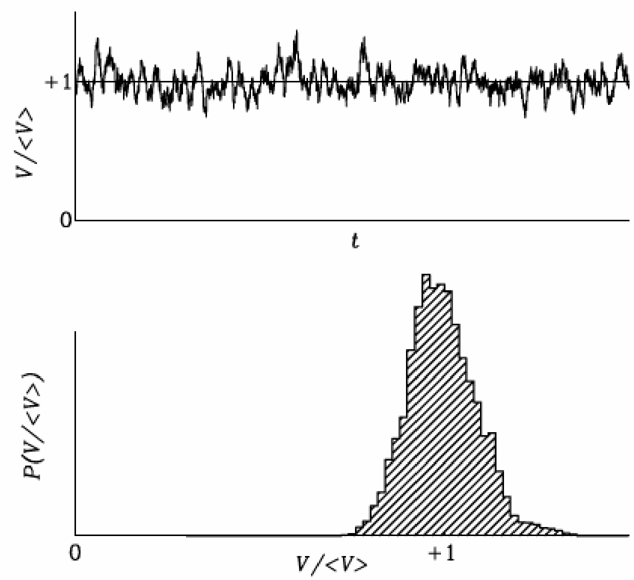



$$T = T_1 + \frac{T_2}{G_1} + \frac{T_2}{G_1 * G_2} + \ldots + \frac{T_N}{G_1 * G_2 * \ldots * G_{N-1}}$$

Cooling the first amplifiers
improves the signal

# Radiometer Equation




http://www.cv.nrao.edu/course/astr534

- **Left**: 50 independent wave trains have been integrated. The expectation value for the voltage is 1 V.
- **Right**: statistical average of 100 wave trains is displayed. The expectation value is much better defined (narrow Gaussian distribution).

# Radiometer equation

$$\sigma_T \approx \frac{T_{sys}}{\sqrt{\Delta\nu \cdot \tau}}$$

- The **limiting sensitivity** of a radio astronomical receiver can be calculated via the **radiometer equation**
- Example continuum radiation: we integrate for 30 seconds the radiation of a source using a bandwidth of 100 MHz. During that integration time, we sample the wave train $30s*100\cdot10^6$ Hz $=3\cdot10^9$. In the very limit we can determine temperature fluctuation of $1.8\cdot10^{-5}*T_{sys}$.
- Example spectral line: we integrate for 30 seconds using a channel-width of 6 kHz, we find $30s*6\cdot10^3$Hz -> $2.3\cdot10^{-3}*T_{sys}$, with $T_{sys}$ = 40 K we derive 92 mK as temperature limit for the line emission.

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# Sensitivity of a radio telescope

# Radiation temperature: Mars

- Radiation temperature from Mars

$$L_{Mars} = \frac{A_{Mars}}{4\pi * (1.52 * 1\text{AE})^2} * L_{Sun}$$

$$L_{Mars} = \frac{A_{Mars}}{4\pi * (1.52 * 1\text{AE})^2} * 4\pi * R_{Sun}^2 * \sigma * T_{Sun}^4$$

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$$4\pi * R_{Mars}^2 * \sigma * T_{Mars}^4 = \frac{A_{Mars}}{4\pi * (1.52 * 1AE)^2} * 4\pi * R_{Sun}^2 * \sigma * T_{Sun}^4$$

$$T_{Mars}^4 = \frac{R_{Sun}^2}{4(1.52 * 1AE)^2} * T_{Sun}^4$$

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$$T_{Mars}^4 = \frac{R_{Sun}^2}{4(1.52 * 1AE)^2} * T_{Sun}^4$$

$$T_{Mars} = \sqrt[4]{\frac{(6.95 \cdot 10^8 \mathrm{m})^2}{4(1.52 * 1.496 \cdot 10^{11} \mathrm{m})^2} * (5800\mathrm{K})^4}$$

$$T_{Mars} = 226\mathrm{K}$$

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# Radiation Temperature to Flux

- Mars has an angular extent of 18“. What is the radiation power at 18 GHz?

$$S = B_\nu * \Omega = \frac{2\mathrm{k} * T * \nu^2}{\mathrm{c}^2} * \pi * (R)^2$$

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# Radiation Temperature to Flux

- Mars has an angular extent of 18“. What is the radiation power at 18 GHz?

$$S = B_\nu * \Omega = \frac{2\mathrm{k} * T * \nu^2}{\mathrm{c}^2} * \pi * (R)^2$$

$$S = \frac{2 * 1.38 \cdot 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}} * 226\,\mathrm{K} * \left(18 \cdot 10^{9}\,\mathrm{Hz}\right)^2}{(3 \cdot 10^{8}\,\frac{\mathrm{m}}{\mathrm{s}})^2} * \pi * \left(9'' * 4.84 \cdot 10^{-6}\,\frac{\mathrm{rad}}{['']}\right)$$

$$S = 13.38\,\mathrm{Jy}$$

$$1\mathrm{Jy} = 10^{-26}\,\frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{Hz}}$$

# Radiation Temperature to Flux

- Size matters! Brightness temperature and flux for a 4-m dish

$$A_{eff} = \frac{P_{received}}{S_{source}} = \frac{2 * \mathrm{k} * T[\mathrm{K}]}{S_{source}[\mathrm{Jy}]}$$

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$$\frac{T[\mathrm{K}]}{S_{source}[\mathrm{Jy}]} = \frac{\pi(2\mathrm{m}*0.7)^2}{2*1.38\cdot10^{-23}\left[\frac{\mathrm{J}}{\mathrm{K}}\right]}*10^{-26}\left[\frac{\mathrm{J}}{\mathrm{m}^2}\right]$$

$$\frac{T[\mathrm{K}]}{S_{source}[\mathrm{Jy}]} = 4.55\cdot10^{-3}\left[\frac{\mathrm{K}}{\mathrm{Jy}}\right]$$

1.39 100-m

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# Brightness Temperatur: Kelvin per Jansky

- What is the brightness temperature of a 1 Jy source received by a 4-m dish?

$$A_{eff} = \frac{P_{received}}{S_{source}} = \frac{2 * \mathrm{k} * T[\mathrm{K}]}{S_{source}[\mathrm{Jy}]}$$

$$\frac{T[\mathrm{K}]}{S_{source}[\mathrm{Jy}]} = \frac{A_{eff}}{2 * \mathrm{k}}$$

$$\frac{T[\mathrm{K}]}{S_{source}[\mathrm{Jy}]} = \frac{\pi(2\mathrm{m} * 0.7)^2}{2 * 1.38 \cdot 10^{-23}\left[\frac{\mathrm{J}}{\mathrm{K}}\right]} * 10^{-26}\left[\frac{\mathrm{J}}{\mathrm{m}^2}\right]$$

$$\frac{T[\mathrm{K}]}{S_{source}[\mathrm{Jy}]} = 4.55 \cdot 10^{-3}\left[\frac{\mathrm{K}}{\mathrm{Jy}}\right]$$

The brightness temperature of Mars received by a 4-m dish is

$$4.55 \cdot 10^{-3}\,\frac{\mathrm{K}}{\mathrm{Jy}} * 13.38\,\mathrm{Jy} = 60.8\,\mathrm{mK}$$

**18.65K 100-m**

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# Detection limits and integration time

$$\Delta T = \frac{T_{Sys}}{\sqrt{\Delta\nu * \tau}} \Rightarrow$$

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$$\tau = \left( \frac{T_{Sys}}{\Delta T} \right)^2 * \frac{1}{\Delta \nu}$$

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# Detection limits and integration time

$$\Delta T = \frac{T_{Sys}}{\sqrt{\Delta\nu * \tau}} \Rightarrow$$

$$\tau = \left(\frac{T_{Sys}}{\Delta T}\right)^2 * \frac{1}{\Delta\nu}$$

A 4-m sub-mm telescope at 100 GHz using a continuum backend with 30 MHz bandwidth and Tsys = 200 K should detect the Mars radiation at 10σ

$$\tau = \left(\frac{200\,\mathrm{K}}{63\,\mathrm{mK}/10}\right)^2 * \frac{1}{30 \cdot 10^6\ \mathrm{Hz}}$$

$\tau = 37\,\mathrm{sec}$ **3.9 $10^{-4}$ sec 100-m**

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# Single Dish vs. Radio Interferometers

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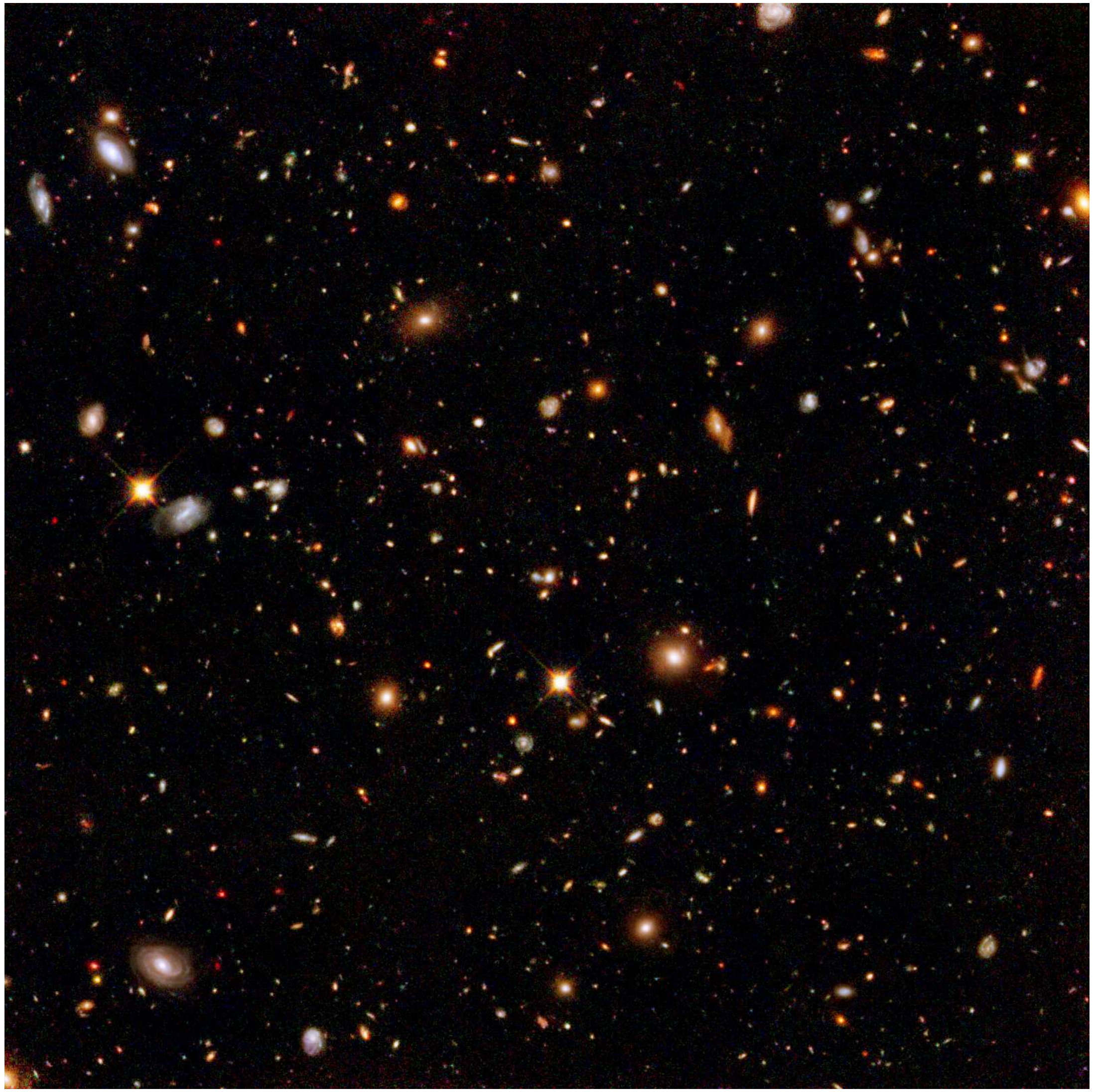
# Confusion limit



http://www.stsci.edu/ftp/science/hdf/hdf.html

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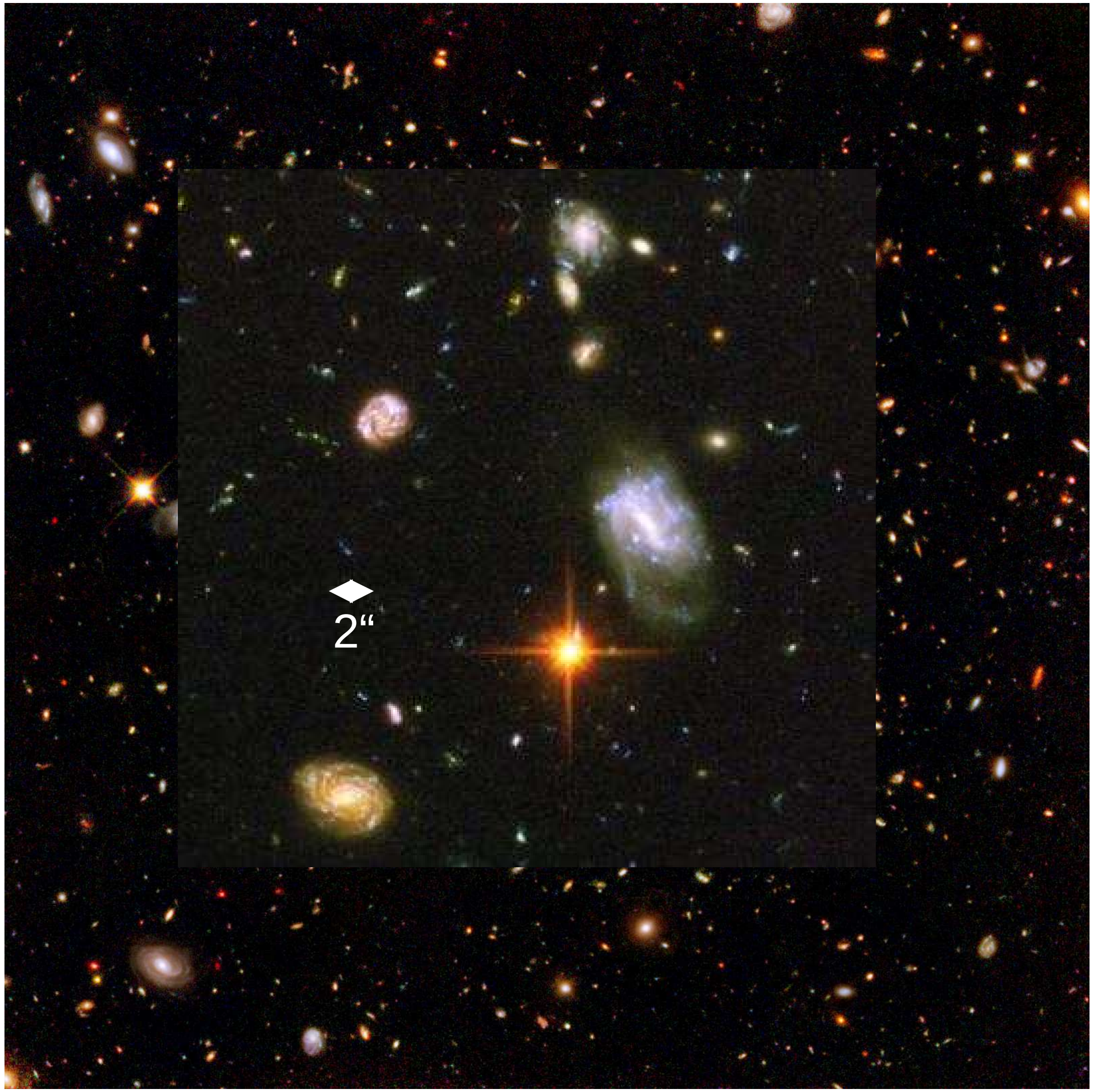


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# Confusion limit

- A telescope with an **angular resolution** of better than **2“** can **watch through the Universe**!
- A **telescope with an angular resolution less than 2“** will finally limited by the superposition of the individual Airy-disks of the objects: it **is confusion limited**
- All single dish radio telescope are confusion limited!
- Continuum maps of single dish radio telescopes do not show up with increasing observing time with fainter details. The „noise“ of these maps is not noise but real structures produced by the emission of objects at cosmological distances
- **Radio interferometers will overcome this limitation**!

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# Why should we care in the „era of arrays“ about single dishs?

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# The value of single dish radio telescopes

- Large single dish radio telescopes offer a huge sensitivity in comparison to radio interferometers.
- The size of the individual radio interferometer dish determines via K/Jy the sensitivity limit of the correlated

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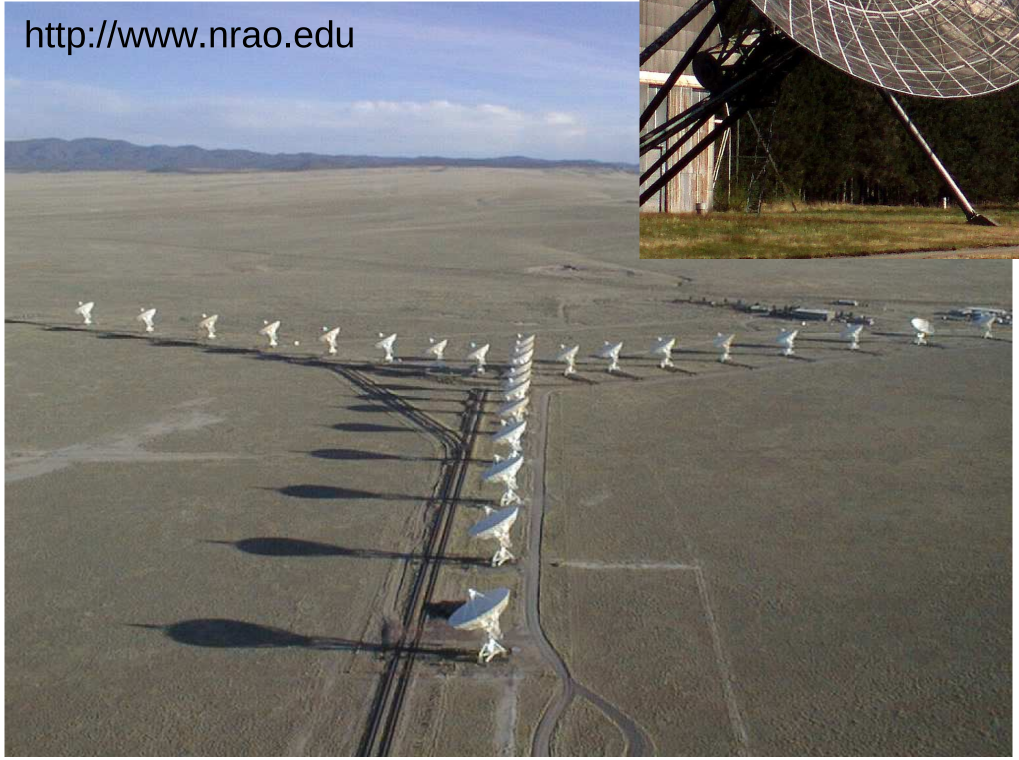


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- A sum of the collecting area of a radio interferometer might be larger than that of a single dish, but its sensitivity is still lower due to the size of the individual dishes

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- The size of the individual radio interferometer dish determines via K/Jy the sensitivity limit of the correlated signal.
- A sum of the collecting area of a radio interferometer might be larger than that of a single dish, but its sensitivity is still lower due to the size of the individual dishes
- Radio interferometer do not allow today to cover a broad range of receivers at different wavelength and multi-feed technology. Expensive receivers (cooled ones) want be standard equipment for radio interferometers but single dishes

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# The value of single dish telescopes

- Large single dishs offer a high number of receivers in their primary or secundary foci

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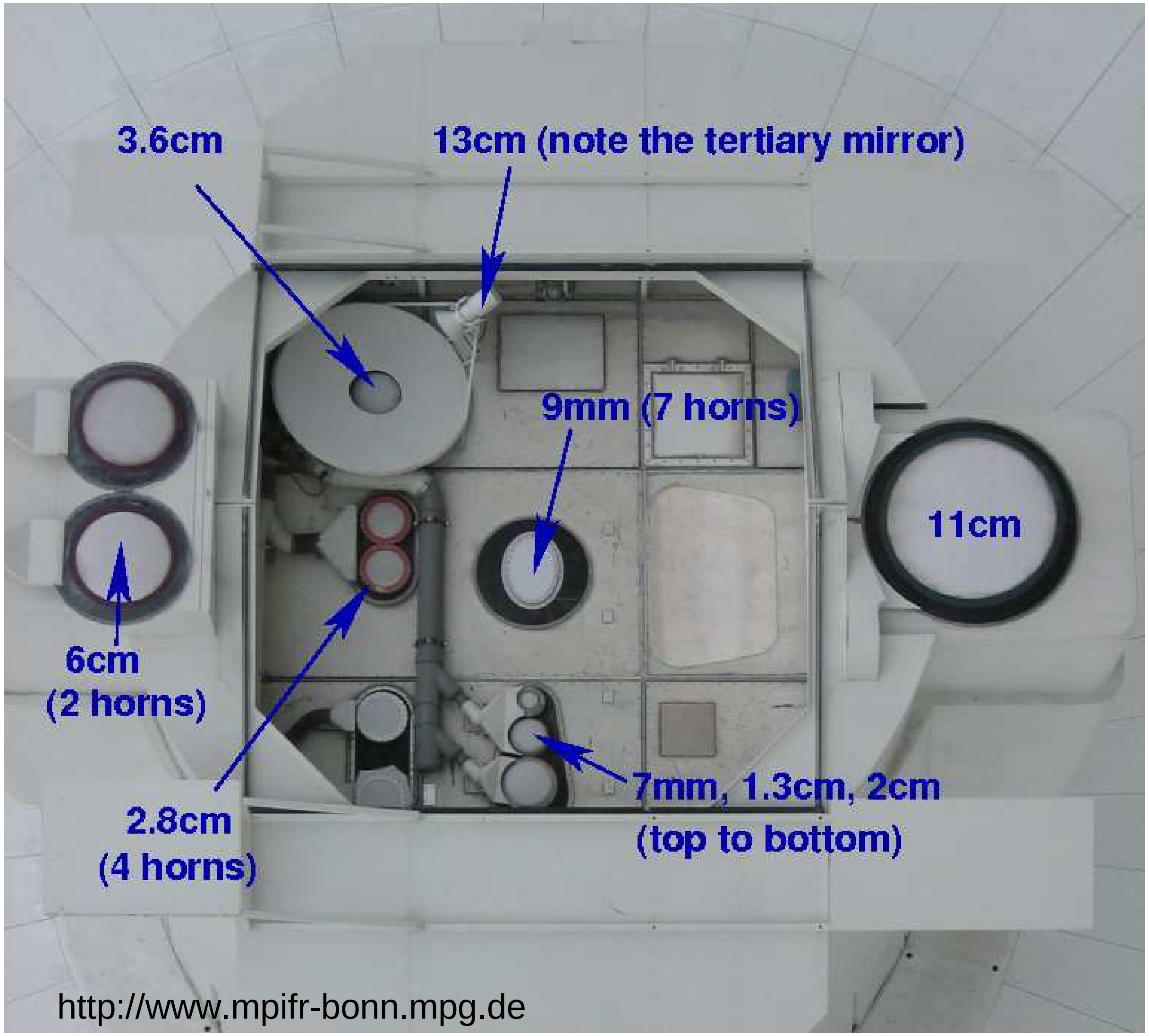
# Effelsberg secondary focus

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  - This allows to measure the spectrum of a source within a few minutes of observing time on mJy level within a single observing run
  - Time variability on scales of days and hours can be tracked accurately
- The image reconstruction of a radio interferometer depends not only on the optics of the radio dishes and the computing power of the correlators but on the UV-coverage!

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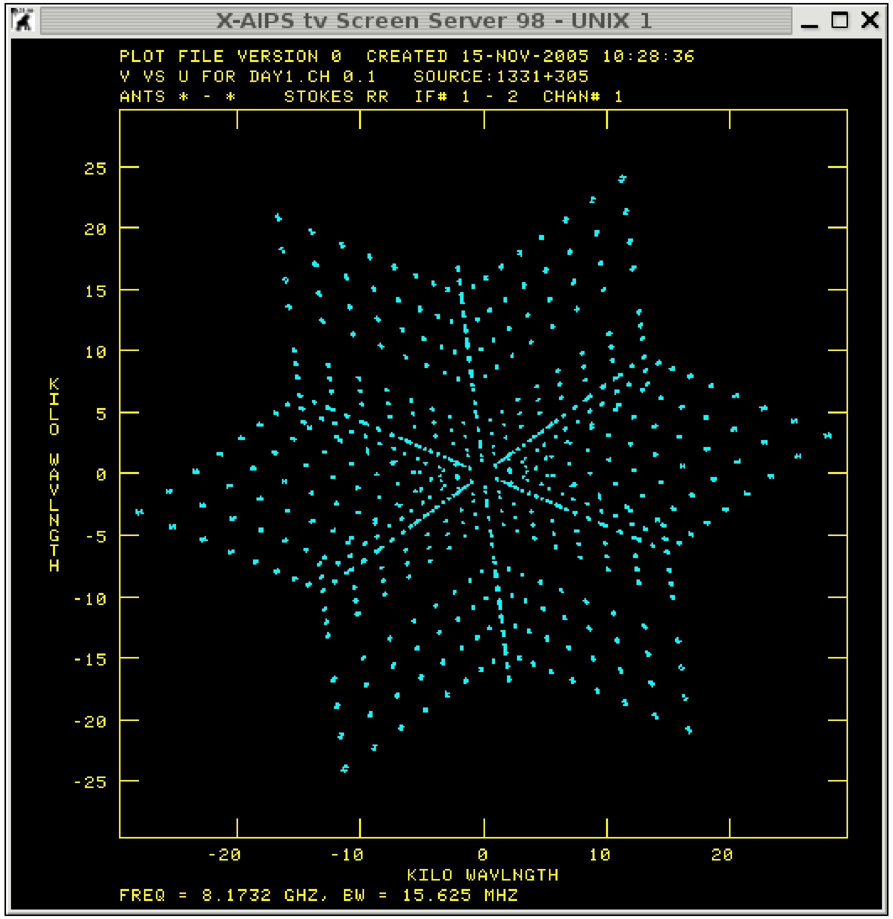
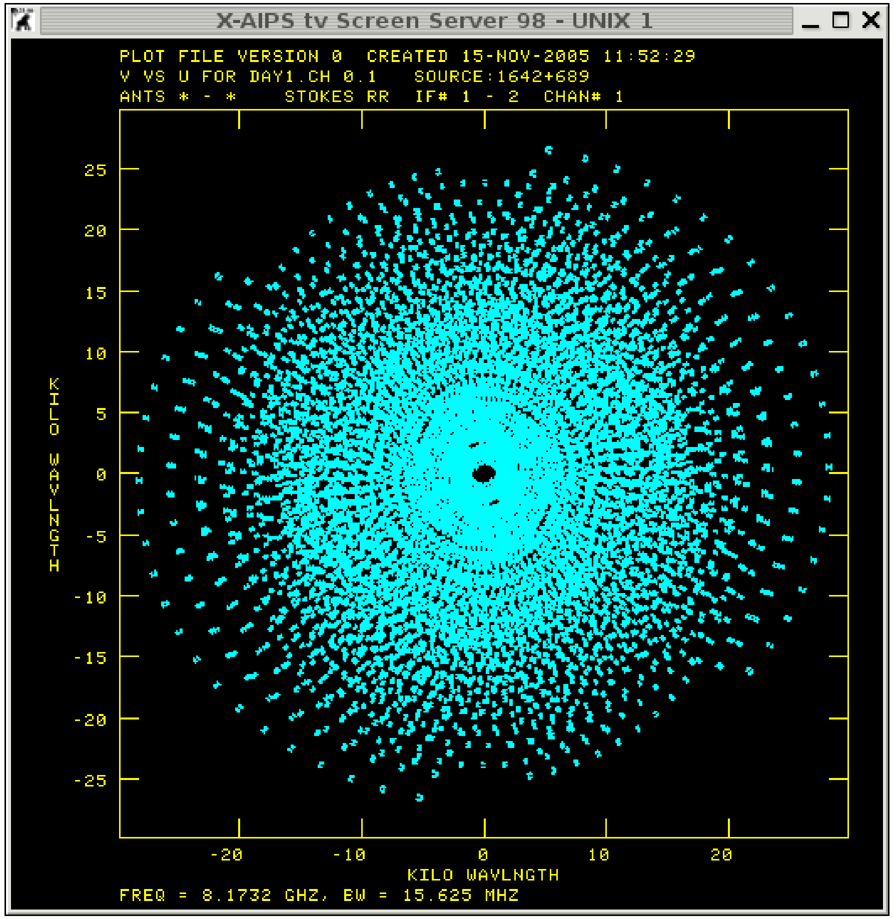
# UV-coverage




http://www.nrao.edu

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http://www.iram.fr

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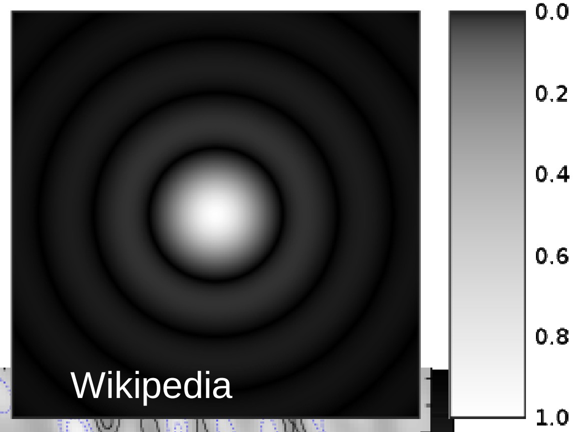
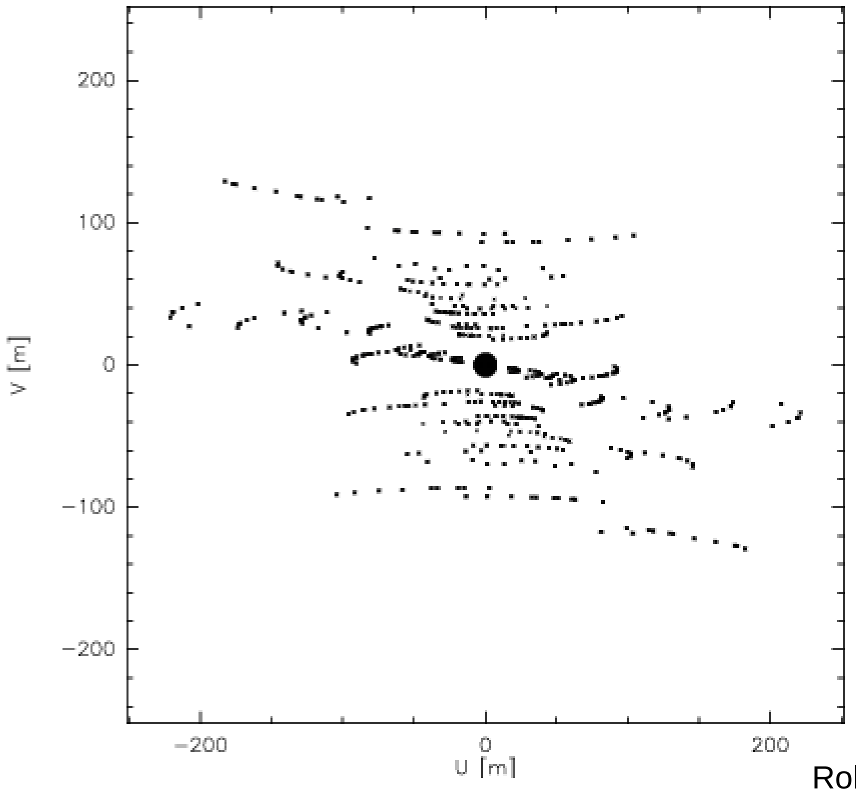
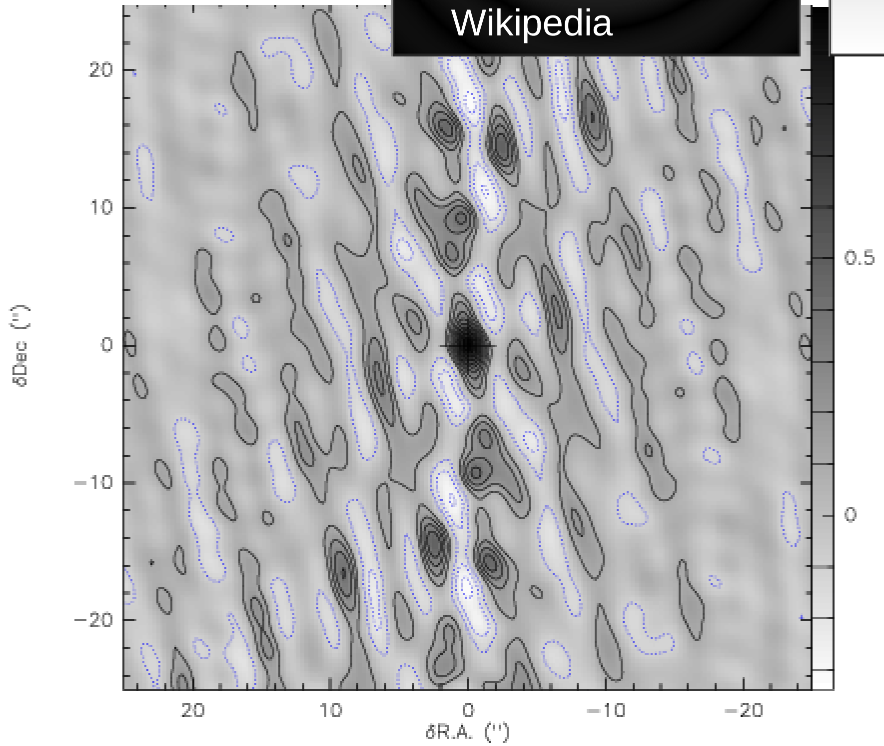
# Point Response Function

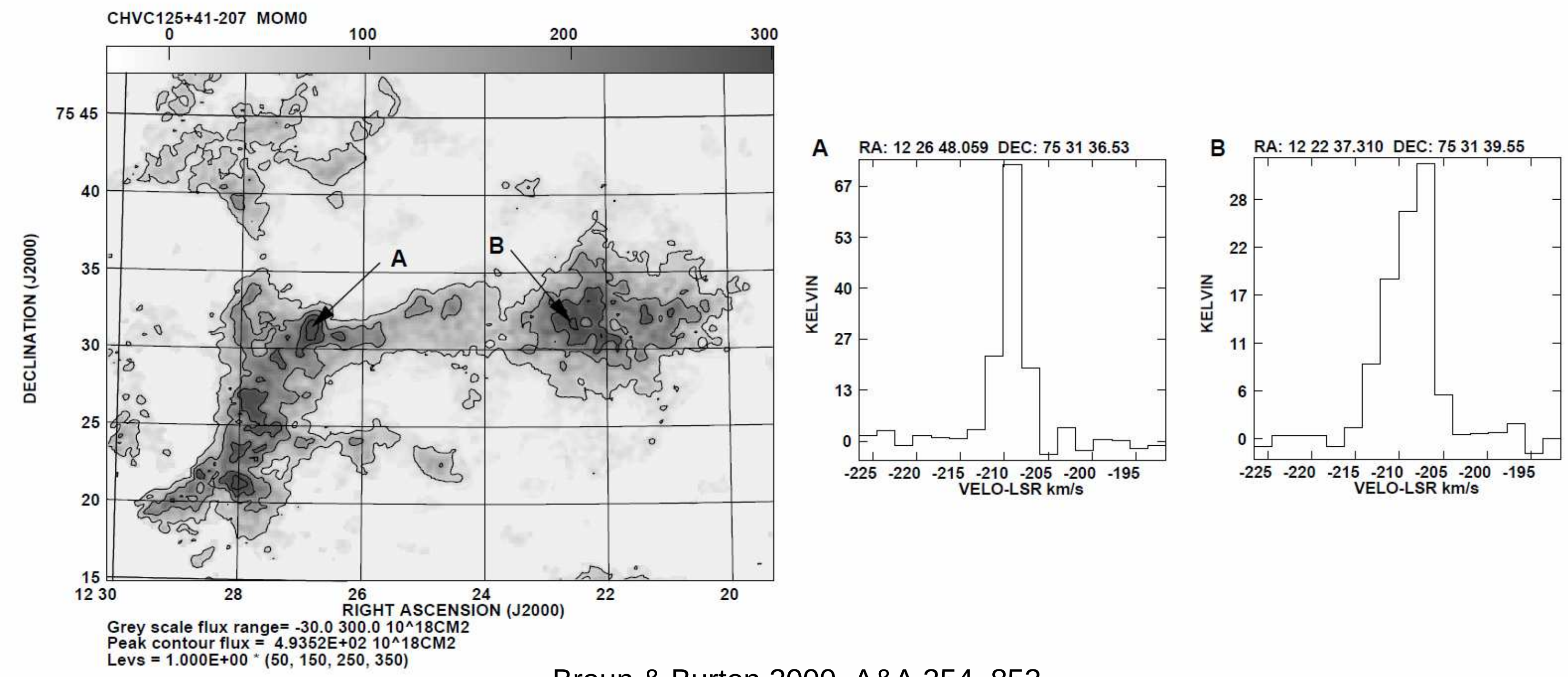



Rolffs et al. 2010

# Selection of science by radio interferometry: an example




Braun & Burton 2000, A&A 354, 853

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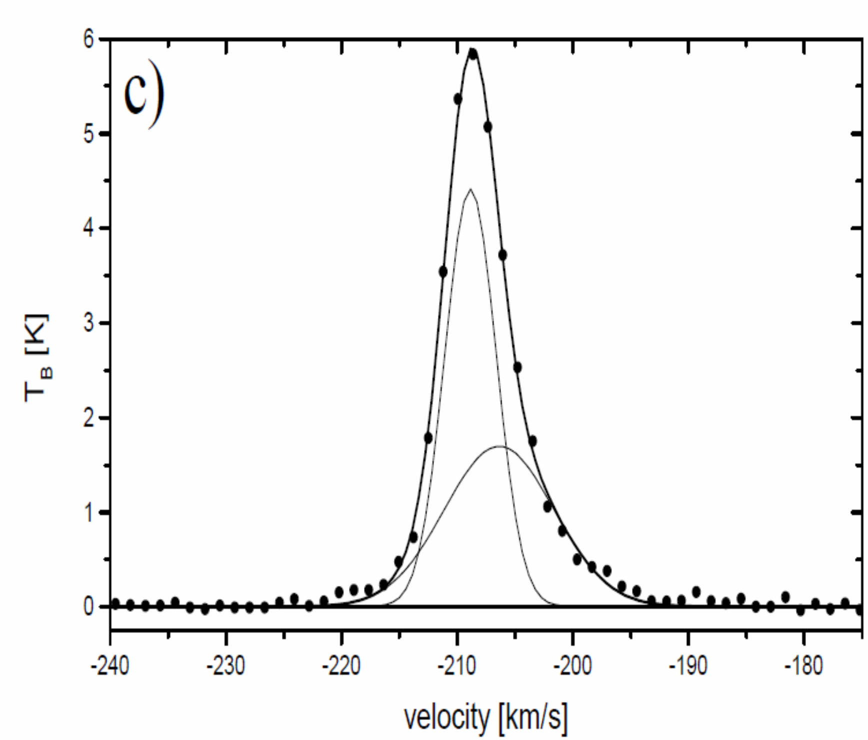
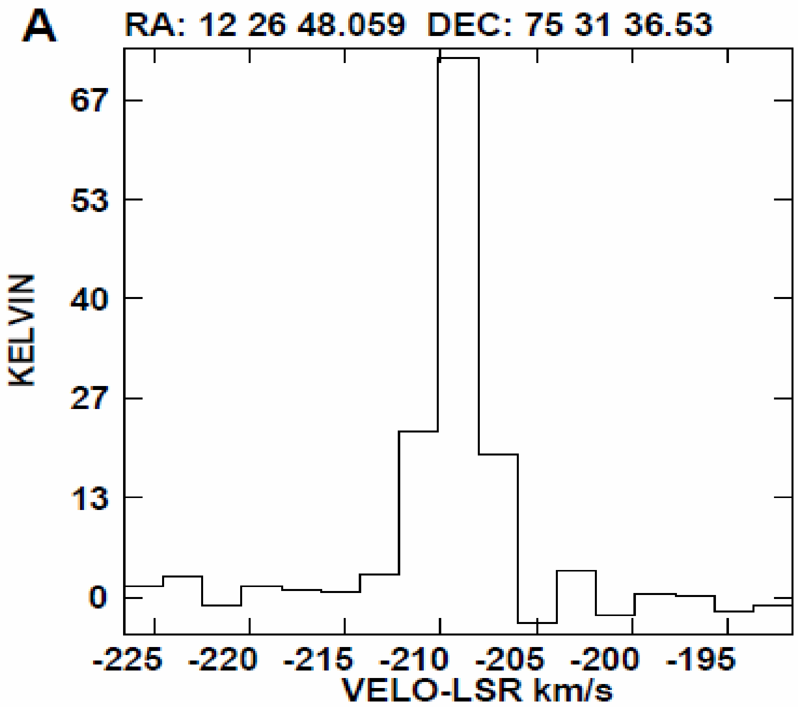
# Comparison on sensitivity




Brüns, Kerp & Pagels 2001, A&A 370, L28




Braun & Burton 2000, A&A 354, 853

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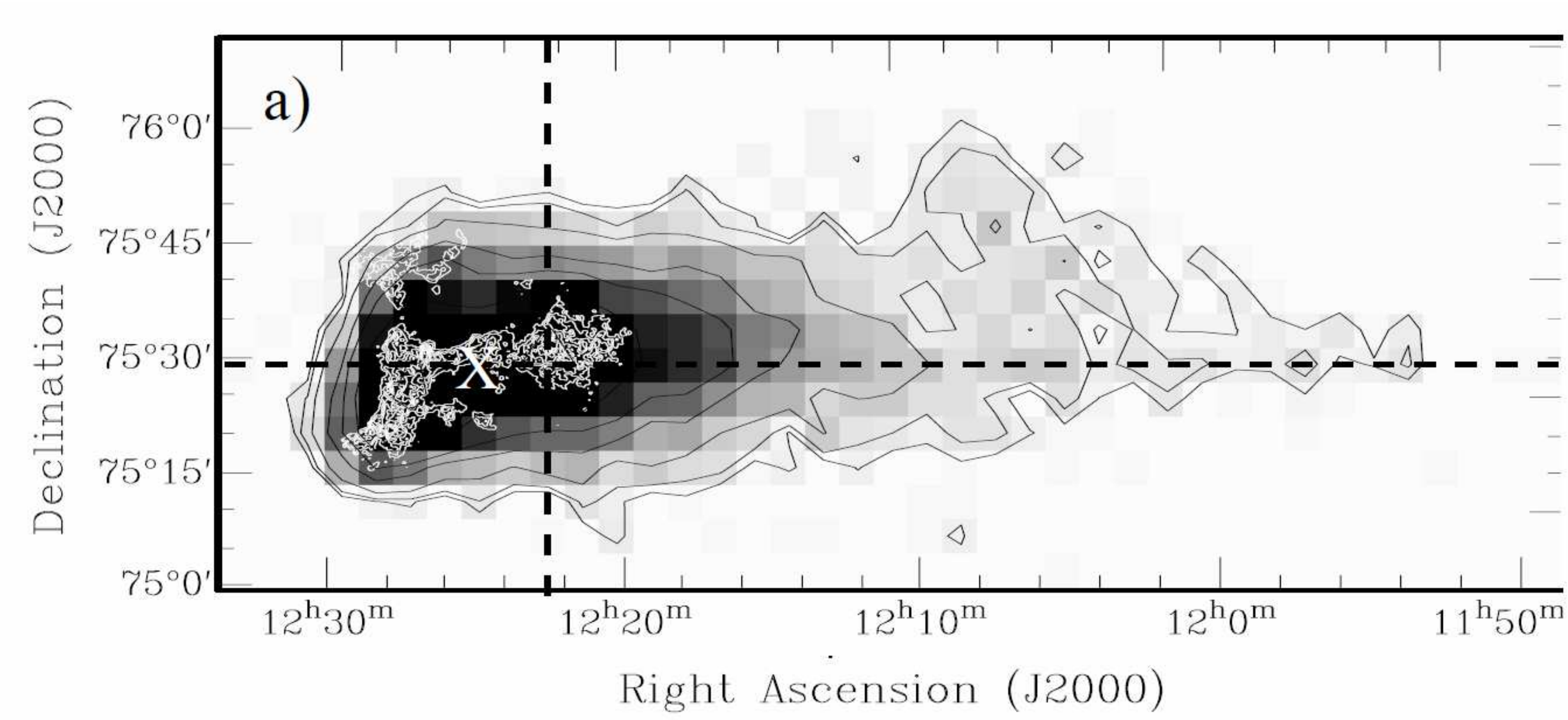


Brüns, Kerp & Pagels 2001, A&A 370, L28

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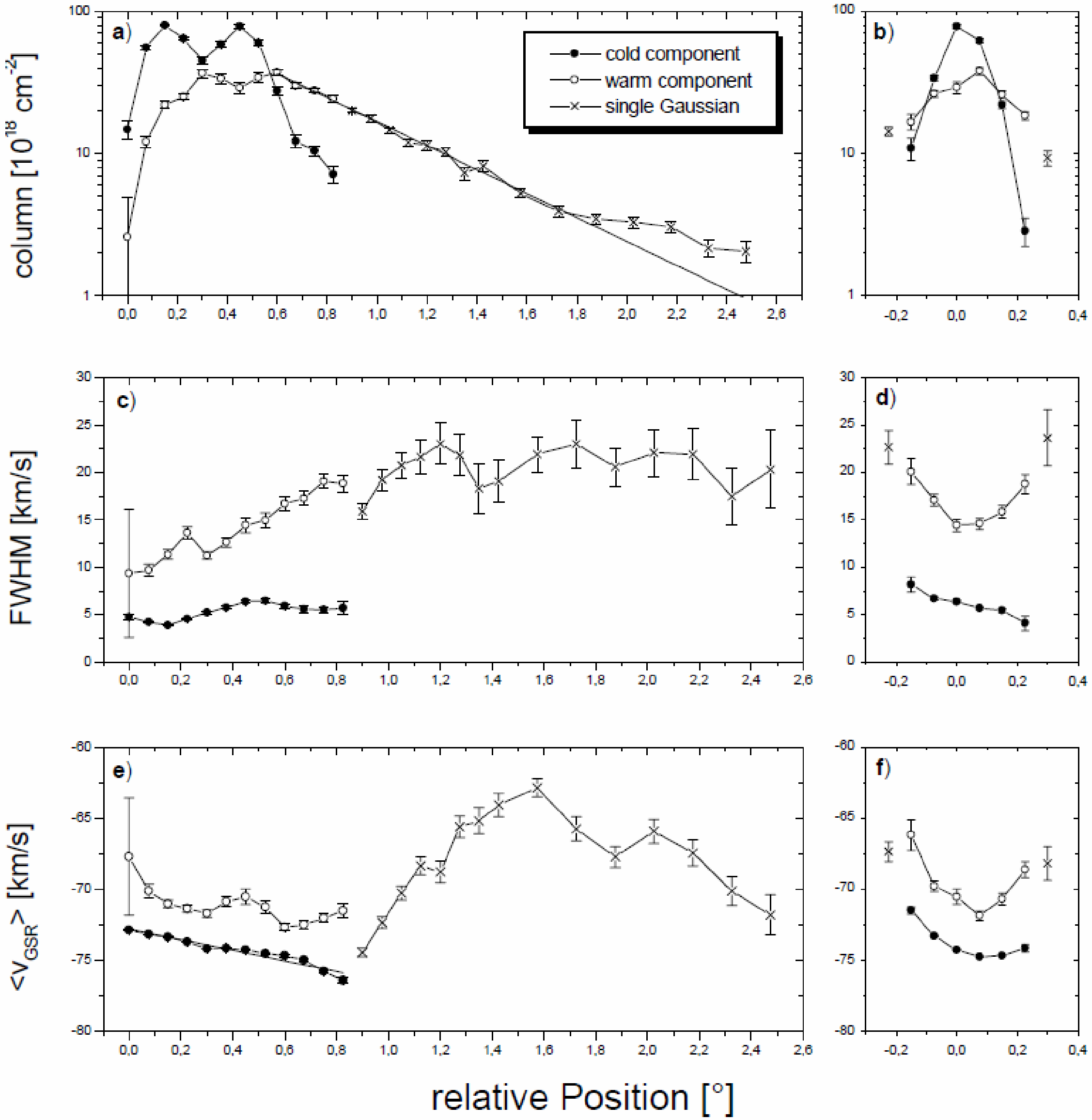
Brüns, Kerp & Pagels 2001, A&A 370, L28

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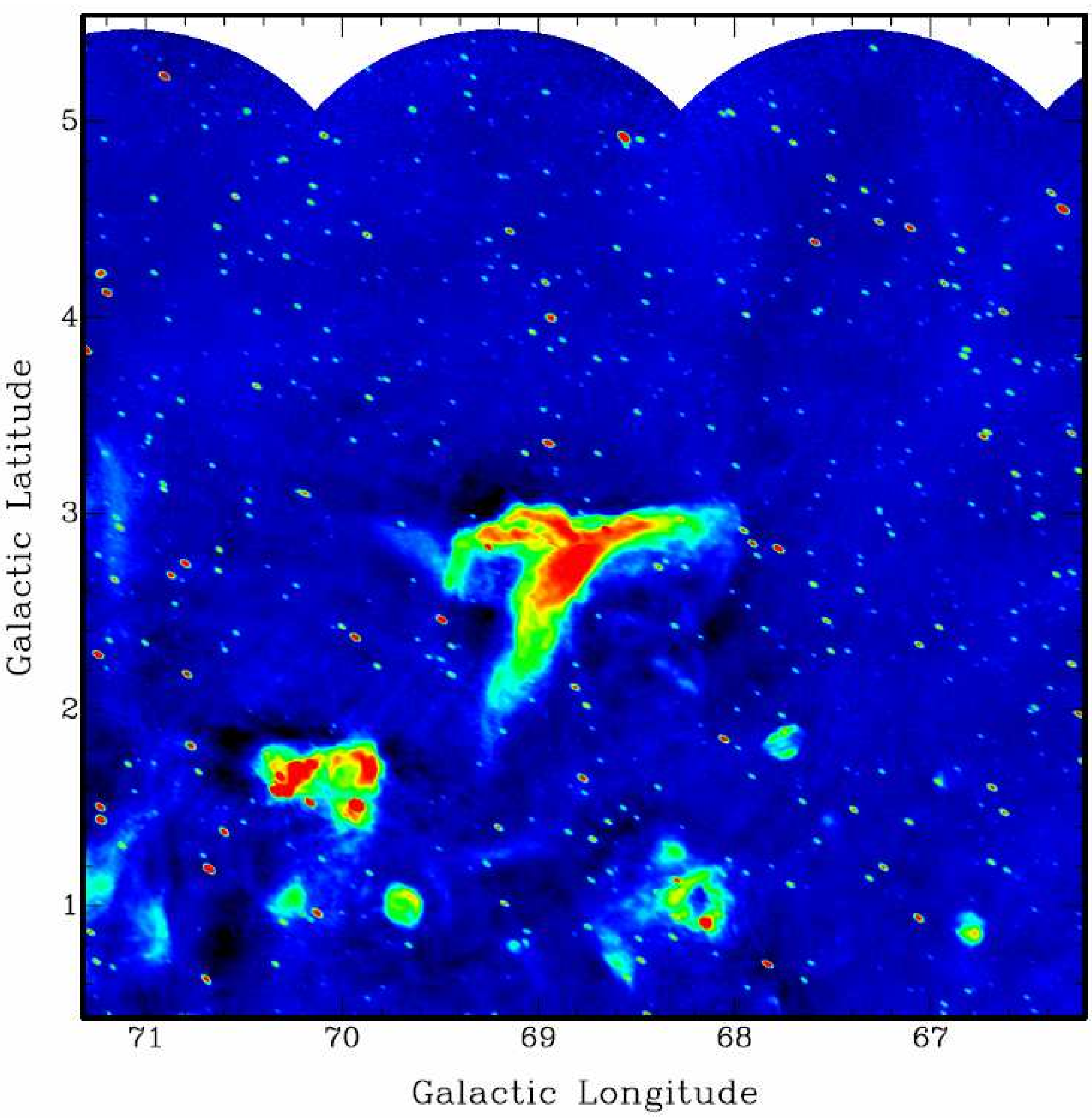




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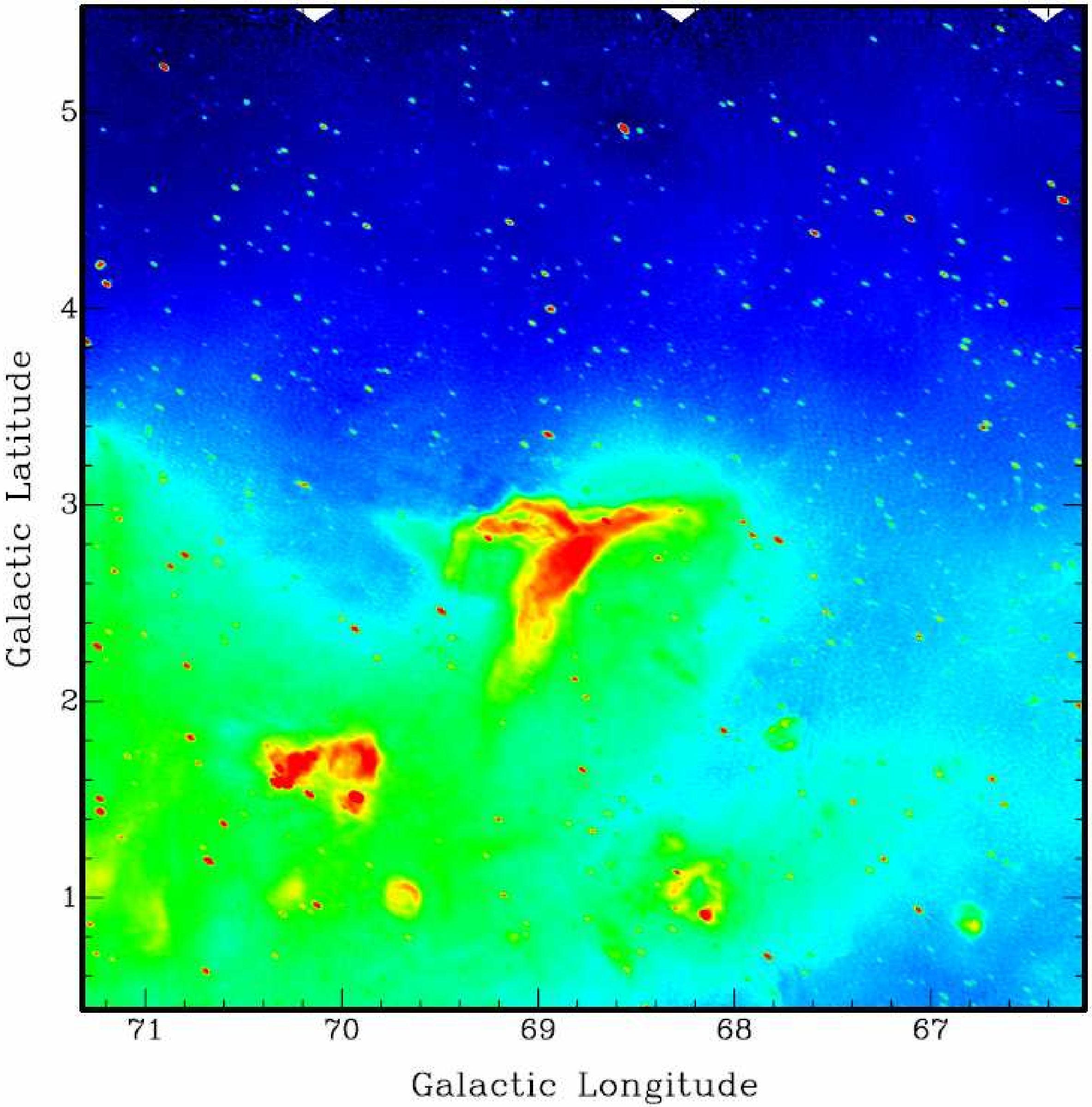




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# Summary

- Single dishes offer highly sensitive radio observations
- Expensive receiver technology can be used
- Multi-frequency observations are feasible due to the high number of receivers in the secondary and prime focus
- Single dish telescope measure large scale intensity distributions while the radio interferometer telescopes are „blind“ for this radiation
- Single dish’s are strongly confusion limited
- The future is the combination of single dish (filled apertures) radio interferometer data (Square Kilometer Array)

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